Quantifying the Penetration Rate of 662keV Gamma Rays through Lead

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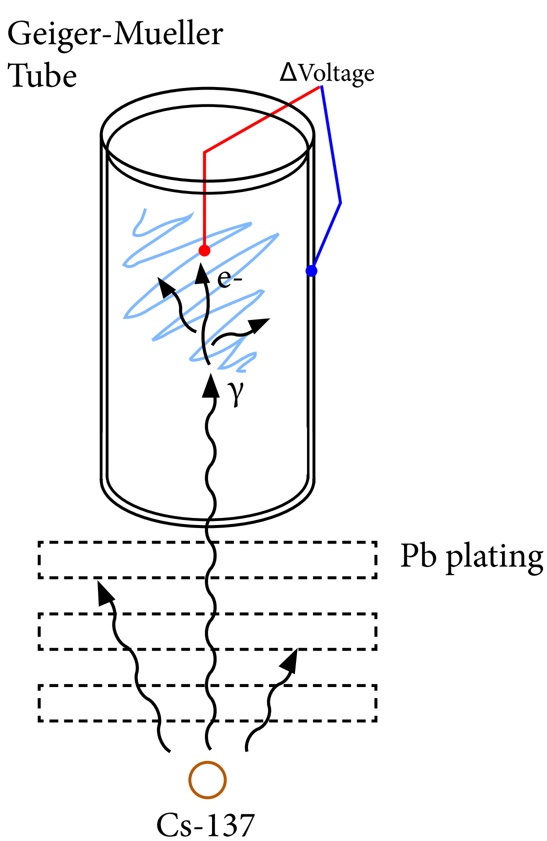
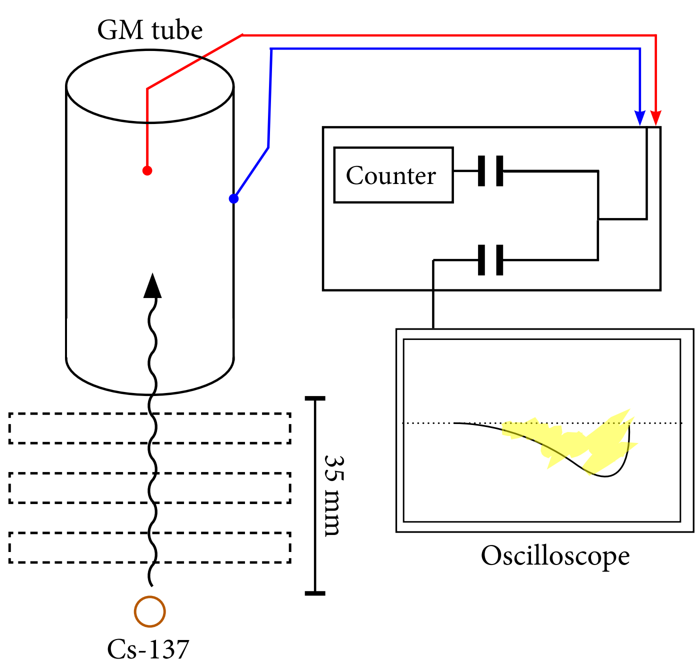
Report by Xavier Boluna
Lab Partner: Scarlet Passer

ABSTRACT:

Using a Geiger-Mueller tube and a Cesium-137 source, we measured the rate of gamma-rays which could penetrate varying thicknesses of lead plating. We analyze the distribution in which we collect counting rate data, the dead-time of our apparatus and electronics, and finally reach a conclusion on the pattern of the intensity of incident gamma rays through lead: $I = I_0e^{\mu x} = e^{Ax}e^{B} = e^{-1.82x}e^{2.55}$ with a X^2 confidence level of 4.88×10^{-2}, suggesting that it is reasonable. As such, we are able to obtain our absorption coefficient value of lead $\mu = 1.82\ cm^{-1} \pm 0.185\ cm^{-1}$ which we then compare to the theoretical value, to find a lacking correlation.





INTRODUCTION:

About thirty times a minute, gamma rays pass through your body. These ionizing electromagnetic waves travel millions of lightyears, scattering their energetic guts across Earth's atmosphere and catalyzing a wider shower of photons through air, living things and solid walls. Gamma rays, in fact, are the most penetrating type of radiation; and likewise the most damaging. Ionizing radiation involves energies capable of damaging organs, mutating DNA and harming other sensitive parts of the human body. It makes sense, then, to figure out what it takes to minimize their penetration rate.

The same gamma rays which travel across our universe can be emitted much closer to home, when unstable nuclei decay and produce high-energy photons, among other particles. Cesium-137 (Cs-137) is a fine example, and the subject of our experiment. We will use increasing thicknesses of lead (Pb) to measure these gamma rays' penetrative abilities.

We measure each decay event with a Geiger-Mueller (GM) tube, as shown in Figure One. As our Cesium decays, it emits gamma rays which meet the air, our lead plates and, occasionally, the interior of our GM tube. Once inside the tube, gas within is energized by the gamma ray, producing increasing cascades of photons and electrons[1]. Maintaining a high electric potential between the anode (red) within the tube and the cathode (wall) ensures that a significant number of electrons from our cascade zips towards the anode, making a blip on our oscilloscope. Each decay event that reaches the GM tube will create a blip in this fashion.

Exactly when a decay happens is independent from the next. The process is essentially random: we don't predict any particular rate of decay; only analyze it over a large timescale.

This random distribution when plotted against probability should follow the Poission Distribution, shown in Figure Two. It follows that for every thickness of shielding we use, the data we retrieve should match the Poisson Distribution in rate frequency.

In the following section, APPARATUS AND PROCEDURE, I will discuss in detail our setup and methods of experimentation. Subsequently,

Figure 1

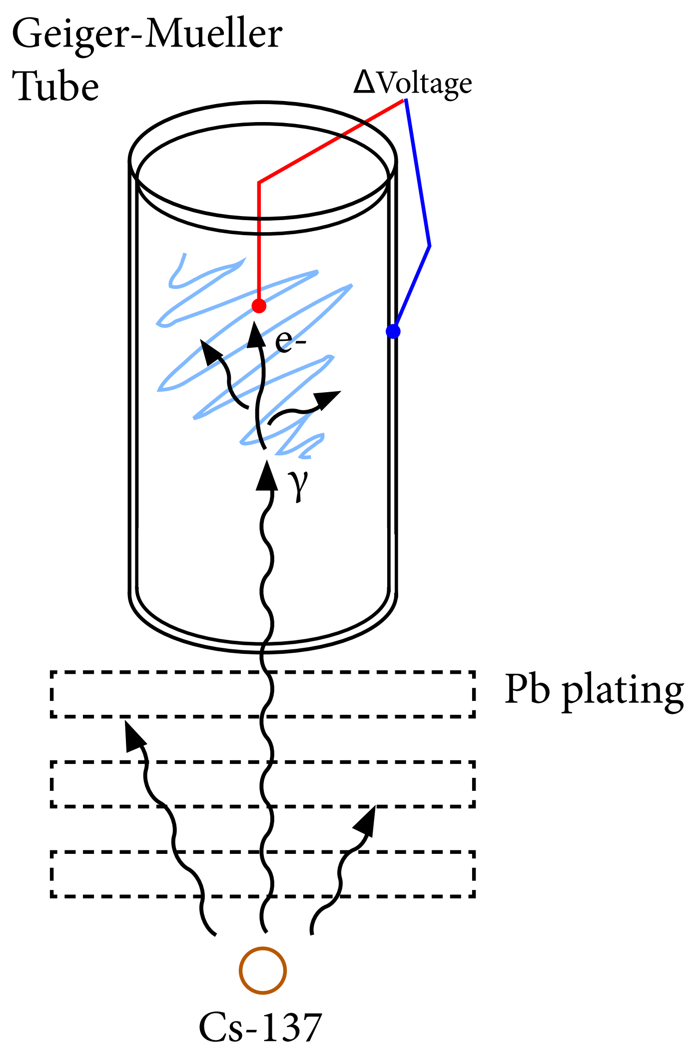


Figure 2

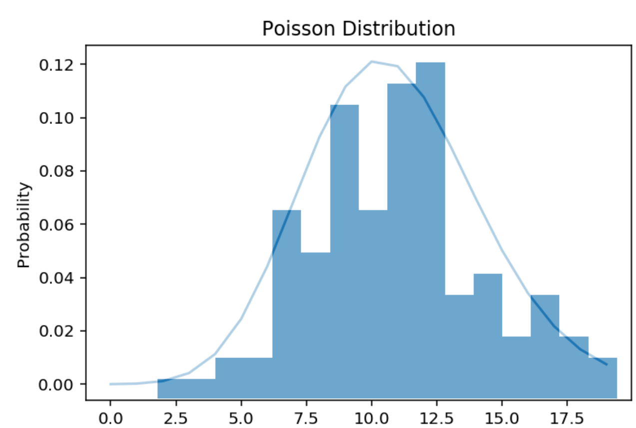


Figure 2 graphs the Poission Distribution obtained from a histogram with real data from an unshielded Cs-137 source from about 30mm away. In this case, it's easy to see that a rate of about 10.8 counts/second is probabilistically the most likely.

[1] Brown, Lab Manual

I will comment and analyze our raw data in RESULTS, including error analysis, and make some experimental measurements and conclusions. Lastly, DISCUSSIONS AND CONCLUSIONS comments on shortcomings of the experiment, offers explanations and justifications for those errors and suggests improvements for subsequent iterations of this experiment. The report is appended by ACKNOWLEDGEMENTS AND SOURCES and the TABLES section, which provides the raw data we used in our experiment.

APPARATUS AND PROCEDURE:

Let's start with a single gamma ray, emitted from our Cs-137 source. We placed our source exactly 4 centimeters away from bottom of our GM tube. As such, our thicknesses of lead plating varied from 3 to 24 millimeters, spanning just over two centimeters. Instead of measuring thickness directly, however, we measured mass and area to calculate more accurately our absorber thickness ρx [g/cm^2].

Our Spectech box houses two capacitors and a counter, and moderates the potential across the anode and cathode of the GM tube. For our experiment, we set the potential at 800 volts. The capacitors ensure that, for each line, the measuring system's own electronics don't interfere with the actual signal.

Therein lies another issue of note: each gamma ray creates an electron cascade, with our output voltage proportional to the number of electrons that are actually activated. Between one cascade and the other, there exists a dead-time where the Geiger tube isn't yet ready to recieve the next signal. Pulses that develop during this relatively short time will not attain their full amplitude. This is a metric defined temporally by when the voltage reaches the 'trigger threshold;' at which the tube registers a pulse at roughly half of the maximum voltage amplitude. We can thereby quantify the dead time by tracing these lines using the tools provided by the oscilloscope. This data is collated further in Table 1.

Figure 3b shows our view of the oscilloscope when we set our pulses on 'infinite persist' in order to make accurate measurements. Measuring the difference in time at the trigger threshold gives us a dead-time value τ = 580μs.

Figure 3a

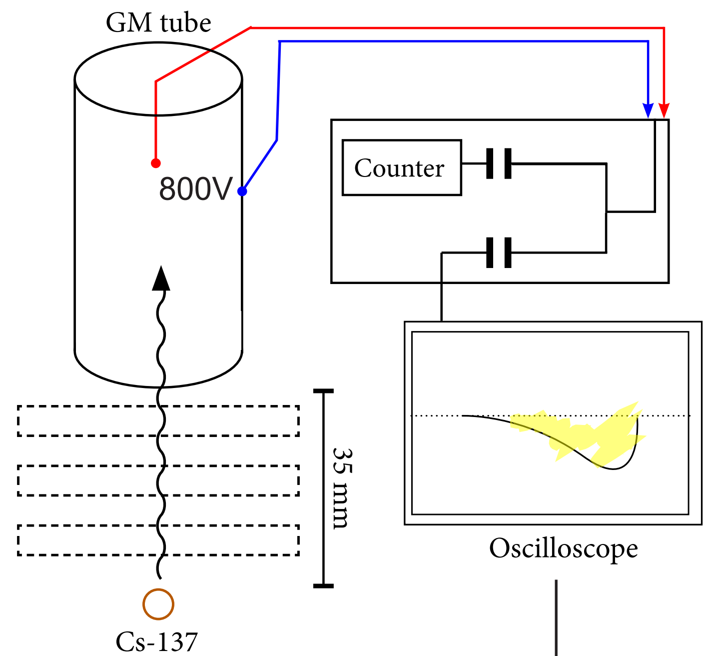


Figure 3b

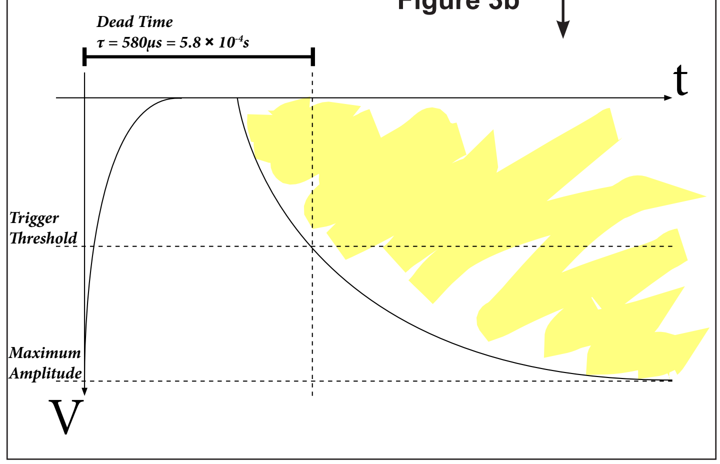


Figure 3a shows our entire circuit setup starting with the Cesium sample which emits a gamma ray into the GM-tube, kickstarting a voltage pulse which is counted and displayed on our oscilloscope.
Figure 3b shows this pulse's waveform, using the trigger threshold (= 1/2 the max amplitude) to determine a dead-time between the GM-tube pulses.

For each lead plate, we obtained a mass and area. We then ran our counter for at least 2,000 counts, and recorded this value and the time it took. Lastly, we took a count of the background radiation (cosmic gamma rays) for the same duration as our longest measurement, 26 minutes and 53 seconds (1,613 seconds).

As such, our independent variables are our area and mass, for which we conveniently calculate absorber thickness, and time. Our dependent variable is the time, with which we calculate our count rate.

[1] Brown, Lab Manual

Both absorber thickness and count rate are quite easily calculated with:[1]

absorber thickness

$$\rho x = m/A \qquad \text{eq. 1a}$$

count rate

$$r = \text{counts/time} \qquad \text{eq. 1b}$$

All this raw data is collected in the TABLES section appended to the end of this report. Calculations from 1a,b are included.

RESULTS

It's first important to mention our background rate. Having taken our source far away from the GM tube, we counted for 1,613 seconds and counted just 814; a background rate of 0.52 counts per second.

We can suggest a model for Intensity (eV), of the exponential decay form[1]:

$$I = I_0 e^{-Ax}$$

$$I = e^{(\text{count rate - bkg rate})} + \text{bkg rate} \qquad \text{eq.2a}$$

which, linearized, gives:

$$\ln(I - \text{bkg rate}) = \text{count rate - bkg rate} \qquad \text{eq. 2b}$$

Such that we can create Figure Four, which plots this relationship using Table 2.

In order to create a linear regression for these points, we must first propagate the error for these values. We represent error as a function of our operations on our parameters[2]:

for some function q = q(x,y)

$$\sigma_q = \delta q = \sqrt{[\,(\partial q/\partial x \times \sigma_x)^2 + (\partial q/\partial y \times \sigma_y)^2\,]}$$

where error is represented by σ_q.

Our two errors to be propagated for ln(I-bkg rate) are the background rate and counting rate.

We can determine the error in our counting procedure by taking more measurements of the background counting rate, which should be constant over more data and time. We can then take the standard deviation of this data to produce an error. Table 3 contains more collection of background data, with which we produce an error $\sigma_{bkg} = \sigma_{counting} = 3.30\times10^{-2}$. I discuss why this determination of error is accurate later in DISCUSSIONS AND CONCLUSIONS.

Having extrapolated the data, we propagate it for our dataset and apply error bars to the data.

Figure 4

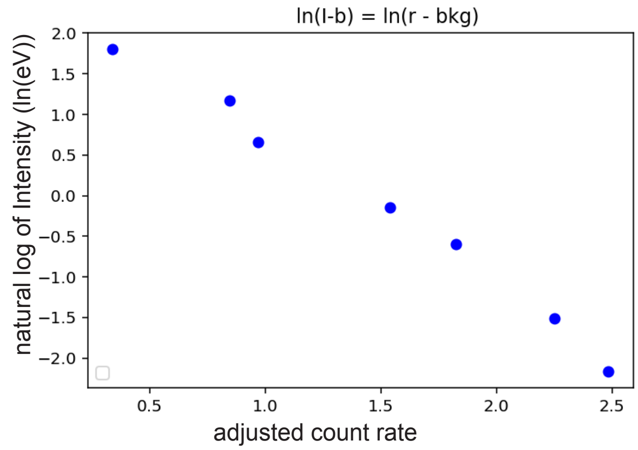


Figure 5

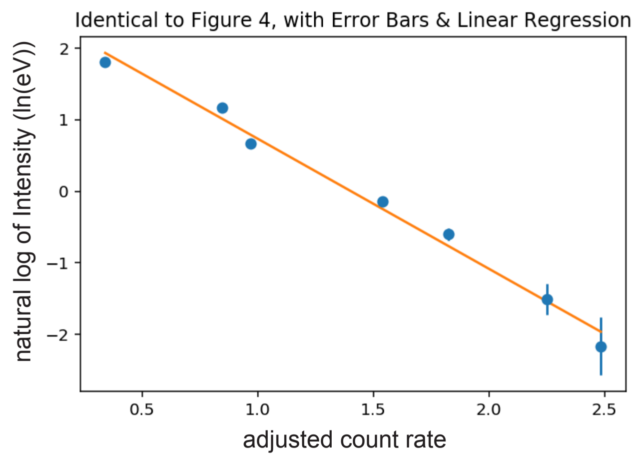


Figure 5 applies our propagated error and plots the weighted least squares regression. The equation is: $y = Ax + B = -1.82x + 2.55$ with a X^2 confidence level of 4.88×10^{-2}, suggesting that it is reasonable.

We determine a weight for each data point using our resulting variance[2]:

$$\omega_i = 1/\sigma_{yi}^2 \qquad \text{eq. 3}$$

which allows us to calculate a weighted least squares regression to apply to our data set[2]:

$$A = (\Sigma\omega x^2\Sigma\omega y) - \Sigma\omega x\Sigma\omega xy)/\Delta \qquad \text{eq. 4a}$$

$$B = (\Sigma\omega\Sigma\omega xy - \Sigma\omega x\Sigma\omega y)/\Delta \qquad \text{eq. 4b}$$

$$\Delta = \Sigma\omega\Sigma\omega x^2 - (\Sigma\omega x)^2 \qquad \text{eq. 4c}$$

where A is our slope and B is our y-intercept.

Figure Five shows the error bars we determined along with the least squares regression we calculated for it.

[1] Brown, Lab Manual, [2]Yan, Statistics Lecture 3

Our next vital step is measuring the degree of confidence we have in our regression. For this, we use the Chi-Square test which takes the form[2]:

$$X^2 = \Sigma_{k=0}\omega_k(O_k - E_k)^2 \qquad \text{eq. 5}$$

where O represents the observed value, E the expected and the result, X^2 indicates:

$X^2 = 0$ :: perfect match
$X^2 \approx n$ :: reasonable
$X^2 >> n$ :: significant disagreement

For our regression, we achieve a Chi-Square value of 0.0488, or 4.88×10^{-2} which suggests a very reasonable regression -- nearly perfect, in fact.

If, then, we decide to take our original equation 2b and raise both sides by e:

$$I - \text{bkg rate} = e^{(\text{count rate - bkg rate})} \qquad \text{eq.2c}$$

we should be able to similarly raise our weighted linear regression by e such that $y = e^{(Ax+B)}$.

Plotting the both of these, we get Figure Six, which aligns with our expectation of an exponential decay.

Breaking down the equation 2c, we attempt to take the limit as $x \to \infty$:

$$I - \text{bkg rate} = e^{(\text{count rate - bkg rate})} = e^{(Ax+B)}$$

$$\lim_{x\to\infty} (I - \text{bkg rate}) = \lim_{x\to\infty} e^{(Ax+B)} = 0$$

This means that the limit of our function should be zero, which matches what Figure Six seems to go towards.

With Intensity - bkg rate = 0, we can determine that at large x, the Intensity is equal to the background rate, which is exactly what we expect if the lead plating is thick enough to block all of the penetrating rays from our source.

The total absorption coefficient, μ, combines the absorption due to the photoelectric, compton and pair production effects[1]:

$$\mu_{total} = \mu_{compton} + \mu_{photoelectric} + \mu_{pair\ production}$$

The total absorption ties into the theory of exponential decay with our original equation 2:

$$I = I_0 e^{-Ax}$$

where we actually reveal[1] that A = μ.

Therefore,

$$I = I_0 e^{-Ax} = (e^B)e^{-Ax}$$

where I_0 is formed by our intercept value.

As such, we make the determination that $\mu = 1.82\ cm^{-1}$ and our initial intensity $I_0 = 2.55$ eV.

Figure 6

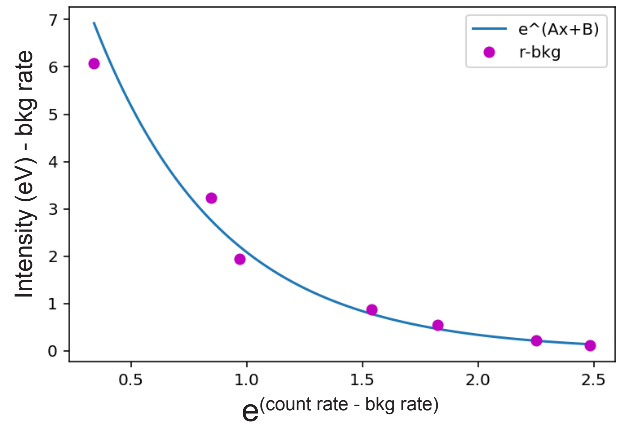


Figure 6 simply rewrites Figure 5 as its more easily recognizable form as an exponential decay, and applies the same operations to its linear regression to create an exponential fit. As we can see, the exponential begins to flatten out, presumably towards the limit zero.

Next, we want to evaluate the uncertainty we obtain in our μ value. We begin by finding the sums of squares and covariance of our data[3]:

$$ss_x = \sigma_x \times n \quad \text{where n is dataset size}$$

$$ss_y = \sigma_y \times n$$

$$ss_{xy} = \Sigma(x_i - \bar{x})\times(y_i - \bar{y})$$

wherein we can evaluate:

$$s = \sqrt{[\ (ss_y - A \times ss_{xy}) / (n-2)\ ]}$$

$$\sigma_A = s/\sqrt{(ss_x)}$$

where σ_A finally signifies our error σ_μ.

We take our $\sigma_y = \sigma_{background}$ from earlier and thusly determine our error to be $\sigma_\mu = 0.185\ cm^{-1}$.

As such, we know our total absorption coefficient $\mu = 1.82\ cm^{-1} \pm 0.185\ cm^{-1} = 1.82\ cm^{-1} \pm 10.2\%$.

[2]Yan, Statistics Lecture 3, [3]Wolfram Alpha, Least Squares Fitting

Our next step with this information is to compare this to our theory. We expect the photon energy emitted from Cesium-137 to be 0.662 MeV, the main photon peak[4]. We can take known theoretical data[1] (which also appears in Table [] at the end of this report) and collate it, as in Figure Seven, to compare our expected photon energy and μ value to determine if it matches the model.

Figure Seven shows lines for both E = 0.662 MeV and μ = 1.82 cm^{-1} ± 0.185 cm^{-1}, which ideally would coincide on or close to the line segments modeled by the data. Unfortunately this does not, which doesn't suggest a good agreement with the accepted theoretical data. The reconciliation for why this data doesn't align perfectly is discussed further in following section.

Figure 7

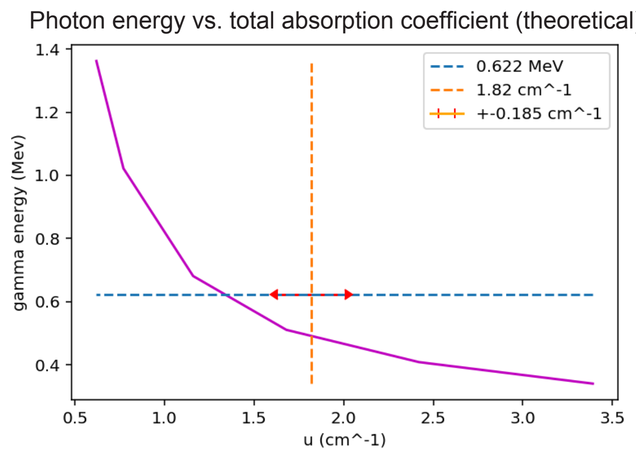


Figure 7 compares the theoretical data of photon energy versus total absorption coefficient and includes the width of data in which μ = 1.85 ± 0.185 occupies. Clearly, the theoretical data doesn't agree with our theoretical data, which is touched on further in DISCUSSIONS AND CONCLUSIONS.

DISCUSSIONS AND CONCLUSIONS:

This last major section is dedicated to various comments needed on the experiment and, largely, to our mistakes: where we went wrong. I will discuss possible remedies to our mistakes and suggestions for future experiments with similar goals.

First, we'll discuss the decision to allow the error for the background to represent the error in the counting rate ($\sigma_{bkg} = \sigma_{counting}$). To do so, we return to Figure 3a, which depicts the circuit which handles our entire setup. The difference between the background and source counting measurements was merely the presence of the source. The source itself, as discussed in the introduction, is inherently random and so cannot have a distinguishable error in and of itself. The electronics alone are the source of error in our measurements for rate. As such, we can assume that the presence of the source does not affect our error, and that analysis of error excluding it is applicable when we do include it.

The second point of note is our discrepancy between our expected μ value and our provided theoretical value. This has a couple implications.

First, it could represent a systemic error in our entire system, given that the value arises from the fitting of our practically raw data. The linear regression is evidently not at fault as our Chi-Square value suggests reasonable accuracy.

Figure 8

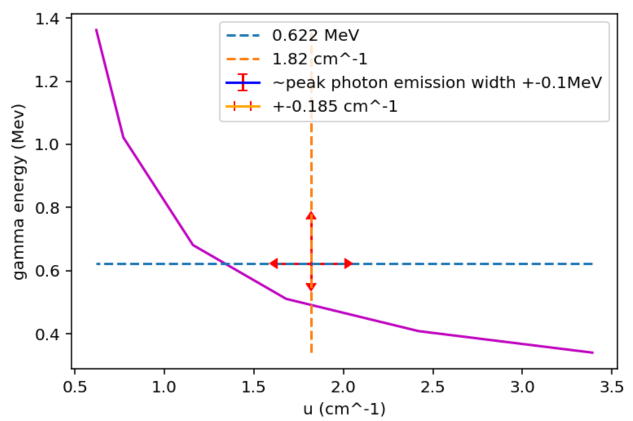


Figure 8 includes a 'error' for a theoretical peak photon width obtained from the Caesium-137 Wikipedia source.[4]

Excluding computational errors, we would still expect our photon emission energy to fall within the relative 'width' of the peak. This data isn't immediately available, but a rough estimate[4] is shown in Figure Eight. Unfortunately, our μ value still does not fit the theoretical curve with the error provided to it.

This suggest a more fundamental issue, with the candidates for our problems being either our electronics and procedure, our early computations or, less-likely, the theoretical values themselves.

As such, similar research is suggested to reconcile these differences, potentially with more data points (thicker slices of lead) or longer spans of time with which to measure counting.

[4]Wikipedia, Caesium-137

ACKNOWLEDGEMENTS AND SOURCES:

This report was created with data performed in an experiment done together with Scarlet Passer.

I received an immense amount of help from both the teaching assistant, Drew Bischel, and Professor of Physics 133, Aiming Yan.

I appreciate the your tireless patience with an experiment that required many wrong turns to make right.

The following texts were used in this report. They are referenced throughout the report in MLA citation format: (Author, Page Number).

1) George Brown et. al., Physical Sciences Department. “Manual for PHYS 133-01”. Lab handbook. University of California, Santa Cruz. 2019. Print.

2) Aiming Yan, “Physics 133 Statistics Lecture 3” . Accessed October 30, 2019

3) Wolfram Alpha. "Least Squares Fitting". http://mathworld.wolfram.com/LeastSquaresFitting.html (accessed November 19, 2019).

4) Wikipedia contributors, “Caesium-137,” Wikipedia, The Free Encyclopedia, https://en.wikipedia.org/wiki/Caesium-137 (accessed November 19, 2019).

TABLES:

Table 1

This first set of data is specifically for the dead-time counting experiment. It involves two sources, gamma (ɣ) and beta (β) rays, and measures their respective rates and dead-times (τ).

Source	counts/time (s^{-1}) = rate (s^{-1})	τ (μs)
ɣ	524/11 = 47.6	550
β	568/7 = 81.1	560
ɣ and β	457/11 = 41.5	580

Table 2

Note that height is a constant 40mm and voltage a constant 800V throughout all the following data points.
For background, we used 841/1613s = 0.521 from Table 3 because it has the longest time duration.

Mass (g)	Area (cm^2)	Thickness (g/cm^2)	counts/time(s^{-1}) = rate (s^{-1})
230	59.61	3.86	1023/155 = 6.60
571	59.61	9.58	1032/275 = 3.75
1166	105.9	11.0	1014/412 = 2.46
1850	105.9	17.5	1009/727 = 1.39
2191	105.9	20.7	1016/949 = 1.07
2704	105.9	25.5	1019/1371 = 0.743
2985	105.9	28.2	1026/1613 = 0.636

Table 3

Note that our last datapoint is the one so frequently mentioned as our overall background radiation; the rest are used to determine error in our counting rate measurements.

counts/time (s^{-1}) = rate (s^{-1})
97/159 = 0.610
164/280 = 0.586
227/413 = 0.550
382/727 = 0.525
495/949 = 0.521
723/1371 = 0.527
841/1613 = 0.521