EXPLORING COMPLEX IMPEDANCE

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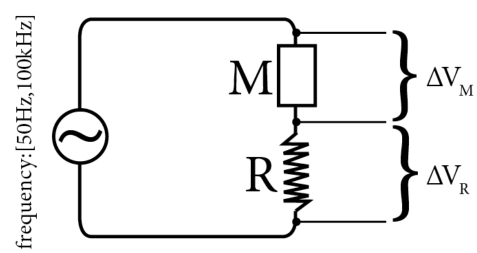
ABSTRACT:

Three 'mystery' boxes contain differing combinations of inductors, capacitors and/or resistors with an unknown possibility of producing resonance. We varied AC frequency input (~) and resistance (R) and measured via an oscilloscope the voltage across the box (ΔV_Z), across the variable resistor (ΔV_R) and the phase between these two signals.

Using known mathematical relationships between electrical impedance (Z) and angular frequency (ω), we matched telltale characteristics and patterns to that of our boxes. Other insights included the relationship between phase and angular frequency, and the effect of changing resistance on electrical impedance.



We have thusly identified that Box B is a resistor and capacitor in parallel and fit it its relationship Z^2 vs. ω^2 with a X^2 goodness-of-fit test value of 0.231 < 17, which is considered reasonable. Our tan(phase) vs. ω has a X^2 of 11,600 >> 19 which indicates its fit disagrees significantly.

Box D appears to be a resistor and capacitor in series, which a Z^{-2} vs. ω^{-2} relationship obtains a fit of X^2 = 49.6 ≈> 20, indicating a somewhat reasonable fit. Again, the X^2 of tan(phase) vs. ω^{-1} gives 5,270 >> 22, indicating that it disagrees significantly.

Lastly, Box E appears to be a resonant capacitor and inductor in series. The plot of Z^{-1} against ω requires two regressions on either side of the resonant peak, which left-to-right have X^2 values of 5.63 and 6.86×10^{-2} ≈ 18 respectively, which indicates a very reasonable fit. Its X^2 for tan(phase) vs. ω-ω^{-1} results 1,000 >> 20, which again indicates that it disagrees significantly.

INTRODUCTION:

The first instinct of any scientist should be to test hypotheses and theories when possible -- either to provide a baseline for further experimentation or challenge the different conditions and permutations in which these patterns present themselves. As such, our experimentation began first with an understanding of the underlying mathematics which govern complex circuits.

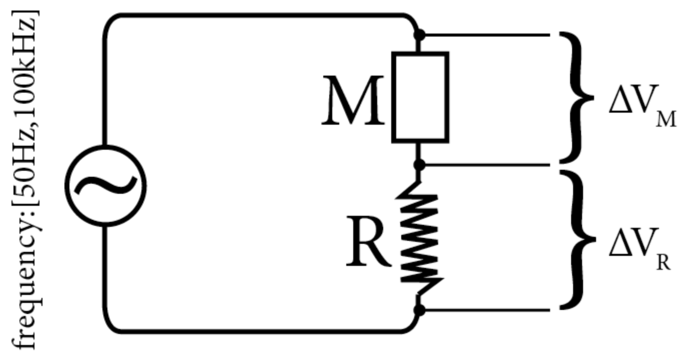
Our goal for this laboratory experiment was to examine circuitry which behaves nonlinearly when comparing impedance with frequency. Impedance is the relationship between oscillating current and voltage; keeping note that these two need not be in phase.

The purpose of our experiment is primarily to identify the 'effective' contents of each of the boxes. This means matching our observed data with a correspondent behavior represented by one of the above equations. This approach is multifaceted: first by comparing both the impedance and the tangent of phase against angular frequency. Second, we examine behaviors at boundaries -- say, a change in resistance -- and watch how correspondent effects in different parameters indicate one circuit type or another.

Our secondary goal is to attempt to calculate the exact values of these constituent components -- the capacitance of one or the inductance of another. It may be inherently difficult to calculate these apart from another, in which cases we will represent these values as a grouped term e.g. "ωC - 1/(ωL)".

The first section, APPARATUS AND PROCEDURE, describes in detail our setup and methods of experimentation. RESULTS comments on and analyzes our raw data, including error analysis, and draws conclusions as to the contents of each boxe and their properties. Lastly, DISCUSSIONS AND CONCLUSIONS reviews the uncertainties and errors made throughout the experiment, offers explanations and justifications for those errors and suggests improvements for subsequent iterations of this experiment. The report is appended by ACKNOWLEDGEMENTS AND SOURCES and the TABLES section, which provides the raw data we used in our experiment.

Figure 1



APPARATUS AND PROCEDURE:

As specified in the introduction, we are hoping to observe specifically three values: voltage across the box (M), voltage across the resistance box and the difference in phase between these two AC signals. In our data tables, five parameters are recorded -- the additional two being frequency and set resistance.
We used the oscilloscope's frequency listing instead of the signal generator's label; though we found little discrepancy. We verified the working order of the resistance box using an ohmmeter.

Our process begins with an AC waveform from the signal generator (~). We varied its frequency into the system from about 50Hz to 100 kHz. The signal travels clockwise in reference to Figure One to the mystery box (M), and then on to the resistance box (R). On either side of the boxes, terminals lead to an oscilloscope. Both channels share the same ground, so we inverted the voltage waveform across the resistance box (ΔV_R) to match the current's clockwise direction.

Before experimenting on each mystery box, we tested its resistance with an ohmmeter. We made sure to record the resistance readings of each of the mystery boxes.

When dialling in the signal generator's frequency, we calibrated the frequency using the oscilloscope rather than the generator's labelling. The oscilloscope also provided the voltages across both boxes and the phase difference between the two waveforms. All of these we recorded in their respective units and order to be viewed in the TABLES section.

RESULTS:

Data is presented in the TABLES section (at the end of this report) in alphabetical order, however it may be informative to know that data was collected chronologically in the order Box E, Box D and Box B.

As discussed in section one, our independent variables are frequency (f) and resistance (R). The correspondent dependent variables are the voltage across the mystery box (ΔV_M), across the resistor ($\Delta V_{R)}$ and the phase (φ) between each signal.

Box B is presented in two sets which are given distinction from each other because they highlight different characteristics of the same box. The first set varies resistance while the second does not. Their implications are further elaborated upon in the analysis of Box B later in this section.

It's informative to first begin with an equation most everyone is familiar with:

$$V = I \times R \qquad \text{eq. 1}$$

(voltage = current × resistance)

Though most of the mathematics we use will resemble this equation, their relationships with frequency are quite different.

One of the more comfortable examples is that of a resistor and capacitor in parallel.

Impedance, which we can describe as the letter Z, is obtained by the equation Z = R + jωl = $|Z|e^{(j\theta)}$ where ω represents angular frequency:

$$\omega = 2\pi \times f \qquad \text{eq. 2}$$

As Z has a complex element, j = √(-1), we evaluate |Z| = √(Z×Z) for our amplitude.

Oftentimes, it is more useful to represent Z as its inverse, Y, which stands for complex admittance:

$$Y = 1/Z \qquad \text{eq. 3}$$

In our case[1]:

$$|Z|^{-2} = 1/R^2 + (\omega^2)(C^2) \qquad \text{eq. 4a}$$

where C is the capacitance. Additionally,[1]

$$\tan(\varphi) = \omega CR \qquad \text{eq. 4b}$$

where φ represents phase.

Other relevant relationships which will be examined in this report include that of the resistor and capacitor in series and the series resonant circuit.

The first is given by the equations[1]:

$$|Z|^2 = R^2 + 1/(\omega C)^2 \qquad \text{eq. 5a}$$

$$\tan(\varphi) = -1/R\omega C \qquad \text{eq. 5b}$$

Whereas the second, a series resonant circuit, is given by the equations[1]:

$$|Z|^2 = R^2 + (\omega L - 1/\omega C)^2 \qquad \text{eq. 6a}$$

$$\tan(\varphi) = (1/R)(\omega L - 1/\omega C) \qquad \text{eq. 6b}$$

where L represents inductance.

One of our primary goals is to match these equations to each of our boxes. Each parameter we collect has an inherent uncertainty which must be factored in to view the data as realistically as possible.

Frequency, which is controlled independently and verified using our oscilloscope to an order of 10^{-3} we consider to have negligible uncertainty. Otherwise, the remaining variable are considered to have inherent uncertainties of:

R: ±5Ω **ΔV_m: ±0.5V**

ΔV_R: ±0.5V **Δphase: ±10**

I discuss our decision-making process for these values in DISCUSSION AND CONCLUSIONS.

It becomes necessary when performing operations on the data to propagate error:

for some function q = q(x,y)

$$\sigma_q = \delta q = \sqrt{[\,(\partial q/\partial x \times \sigma_x)^2 + (\partial q/\partial y \times \sigma_y)^2\,]}$$

We perform a weighted least squares linear regression fit using the weight parameter computed from the dependent variable (y-axis):

$$\omega_i = 1/\sigma_{yi}^2$$

Such that slope coefficient A, is[2]:

$$A = (\Sigma\omega x^2\Sigma\omega y) - \Sigma\omega x\Sigma\omega xy)/\Delta \qquad \text{eq. 7a}$$

And intercept coefficient b is:

$$B = (\Sigma\omega\Sigma\omega xy - \Sigma\omega x\Sigma\omega y)/\Delta \qquad \text{eq. 7b}$$

where:

$$\Delta = \Sigma\omega\Sigma\omega x^2 - (\Sigma\omega x)^2 \qquad \text{eq. 7c}$$

to form some linear equation y = ax + b. This method is known as Least Squares Fitting.

With each set of data, we can evaluate its error by obtaining the expected theoretical value of a value (usually Z) and comparing it against the observed value through the Chi-Square (X^2) method[2]:

$$X^2 = \Sigma_{k=0}\omega_k(O_k - E_k)^2 \qquad \text{eq. 8}$$

where n is the degrees of freedom:

$X^2 = 0$ :: perfect match

$X^2 \approx n$ :: reasonable

$X^2 >> n$ :: significant disagreement

where the expected value E_k is derived from our linear regression.

[1] Brown, Lab Manual; [2]Yan, Statistics Lecture 3

BOX B:

For our first figure, we plot impedance $Z = \Delta V_m / I = \Delta V_m \times \Delta V_R / R$. Computing and plotting Table 1b yields Figure 2.

One important note is that our data-collection methods in higher frequencies resulted larger intervals between them. As a result, many points are clustered in the lower frequencies.

For our purposes in Box B, this did not affect the resolution of the analysis. However, in order to space out these early points and better visually demonstrate the data, we occasionally use logarithmic axes.

In order to create a straight line with these points, we take the inverse square of the vertical axis ($|Y|^2 = 1/Z^2$) and the square of the horizontal (ω^2). We must also propagate error from our initial uncertainties. Figure 3 demonstrates this and includes the correspondent line of best fit using equations 7a,b,c.

Of additional benefit is the plot of tan(φ) (in radians) versus angular frequency. Figure Four shows this relationship with error and a line of best fit included. In the case of this plot, the error is a negligible value and thus does not appear.

Having calculated our weight ω_i, we can calculate our Chi-Square goodness-of-fit test using:

$$\mathbf{E_k = regression\ line = A + (B \times x)}$$

The result is a Chi-Square value of 0.231 which falls within [0,n] where n is our degrees of freedom (sample size - constraints = 20 - 3), 17. This value indicates a reasonable fit.

Box B has an interesting history, as mentioned earlier, in that the data was collected in 'two' sets. The first, in which we varied the resistance, and the second, in which we fixed it at 50Ω. Surprisingly, the discrepancies between these two sets allow us our first clue as to the contents of the box. These sets were taken in a continuous two-hour period; essentially two concentrations of the same data set. As such, it does not constitute the deletion of data nor any modification to it. This is touched on more DISCUSSIONS AND CONCLUSIONS. Figure Five depicts this 'initial' set of data, which shares the first nine data points with that of Figure Four (see italicized data in Tables 1a,b).

Figure 2

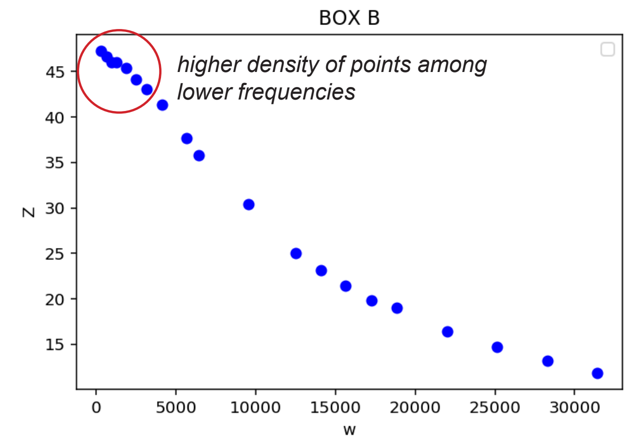
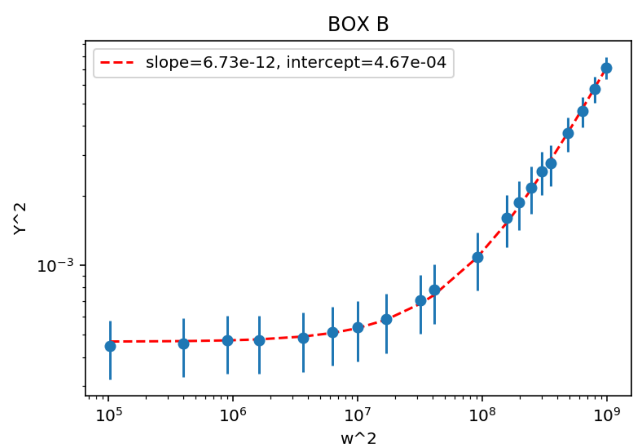


Figure 3
(Logarithmic Axes)



On two axes scaled logarithmically, this plot demonstrates the accuracy of this fit within our error.
$X^2 = 0.231 \approx 17$

As highlighted by the vertical colored lines, there are apparent discontinuities -- jumps in electrical impedance -- which coincide with positive changes in resistance.

Taking a look at the nature of an capacitor, we note that[3] $V_C = V_0e^{-t/RC}$. Disregarding its dependence on time, we notice that $V_C \alpha e^{1/R}$. That is, increasing R will decrease voltage increasingly quickly. This is our first clue that the mystery box contains a capacitor.

Turning back towards the data in Figure Three, we can then begin to match our data to our theoretical mathematical patterns. Having inverted Z^2 as Y^2, we can rephrase equation 4a as:

$$|Y|^2 = 1/R^2 + \omega^2C^2$$

Comparing this to Figure Three, we can see that at small values of ω, Y^2 is small and largely proportional to the constant $1/R^2$. As angular frequency increases, the ω^2 term begins to dominate, shooting Y^2 higher towards the right.

Likewise for equation 4b:

$$\tan(\varphi) = \omega CR$$

We should expect a generally linear relationship between tan(φ) and ω. Figure Four portrays this almost exactly; most notably with a slope on the order of 1/10,000. Our Chi-Square test unfortunately results a $1.16\times10^4 >> 19$, indicating that we have a significant disagreement with our theoretical model. This is because of the outliers that are apparent in Figure Four. I discuss this later in DISCUSSIONS AND CONCLUSIONS.

Still, we have convincing evidence, to believe that Box B is a capacitor in parallel with our resistor box.

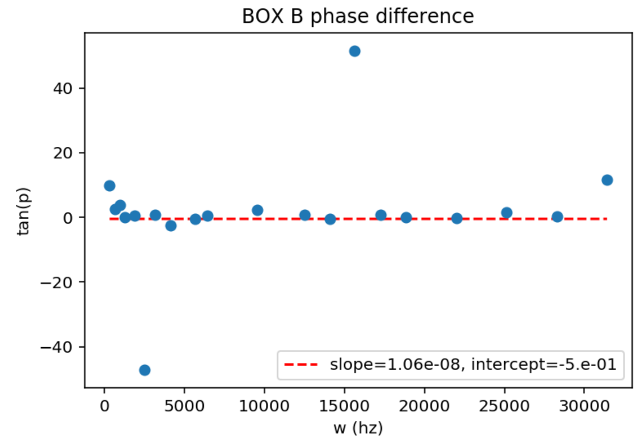
Armed with this information, we can begin to extract data about this capacitor.

Solving for C in equation 4a, we get:

$$C^2 = (|Y|^2 - 1/R^2)/\omega^2$$

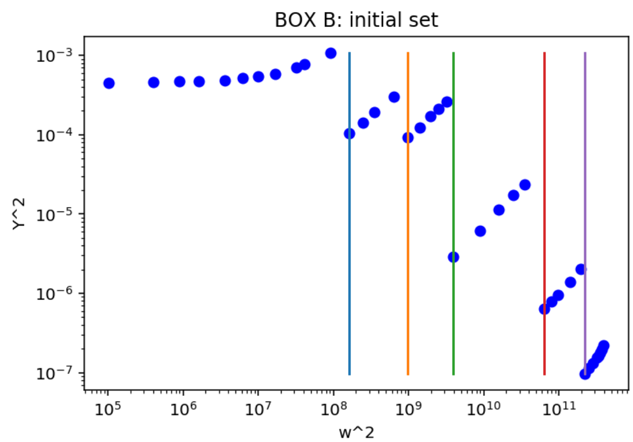
Using this method on our data, we recieve a mean C value 4.90×10^{-6} with a standard deviation 4.49×10^{-6}. A capacitance on the order of 10^{-6} is not uncommon and its standard deviation is likewise negligible.

Figure 4



Notice the outliers which deviate significantly from the best fit line. Error mathematically results to be negligible.
X^2 = 11,600 >> 19

Figure 5



These vertical lines denote a unique event: a change in set resistance. We know that capacitors in parallel produce this behavior.

[3]Wikipedia, RC Circuit

BOX D:

Figure Six plots the data from Table 2 into logarithmic axes Z and ω.

In order to linearize the data in Figure Seven, we plot Z^2 and ω^{-2} and propagate error with it. This graph noticeably has an ill-fitting regression line and seems to involve a linear component whose influence is lesser at small values of ω^{-2}. Our goodness of fit test reveals a value of 49.6 ≈ 20, which indicates that the fit is *nearly* reasonable but not quite.

Moving forward, we can easily see that Z^2 α $1/\omega^2$ and looking back at equation 5a:

$$|Z|^2 = R^2 + 1/(\omega C)^2$$

This equation involves an capacitor and resistor in series where $1/\omega^2$ dominates at small frequencies whereas R^2 does at large ω. Looking at Figure Seven, this description seems to fit the bill -- at small frequencies (right-hand side), the pattern seems largely linear whereas large frequencies (left-hand side) bottom out into a constant.

As the boundary of this transition from $1/\omega^2$ to R^2 is not discretely defined, part of the error in the regression fitted to the lowest frequencies is attributable to interference from both terms of the equation.

The plot of tan(φ) versus ω doesn't yield a much prettier fit. Its Chi-Squared value is 5,270 >> 22; it is not a reasonably linear fit. Based on equation 5b, however:

$$\tan(\varphi) = -1/R\omega C$$

Where tan(φ) is proportional to -1/ω. Figure Eight reflects this pattern, with linear relationship across the graph (keeping note that the horizontal axis is (1/w), however the slope is notably positive rather than negative as the equation suggests. This is partially attributable to the odd spread of points in lower frequencies, which pull and push the data quite a bit. This is also backed up by its poor Chi-Squared value

Frankly, this box's strongest indicative sign towards a capacitor and resistor in series is its Chi-Squared test in Figure Seven, but it fits this pattern best in comparison to any other others.

Pushing forward, we calculate the value of the capacitor in similar fashion to that of Box B using equation 5a:

$$C^2 = 1/\omega^2(Z^2 - R^2)$$

Figure 6
(Logarithmic Axes)

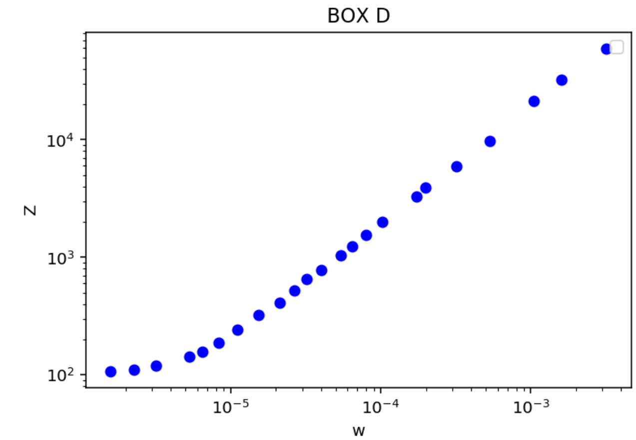
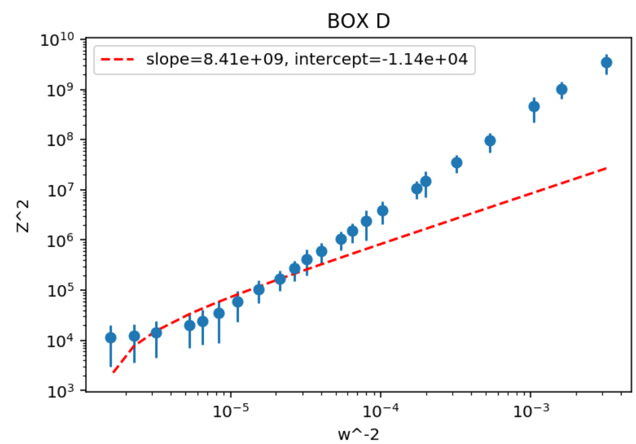
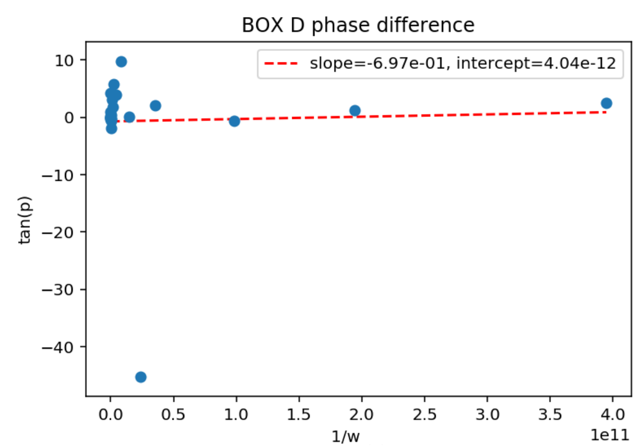


Figure 7
(Logarithmic Axes)



This fit looks odd but is actually somewhat reasonable; possibly a result of the logarithmic axes accentuating small differences.
X^2 = 49.6 ≈> 20

Figure 8



X^2 = 5,270 >> 22

We then obtain a mean value for capacitance C of 7.21×10^{-8}, or about one nanofarad, with a standard deviation of 4.68×10^{-8}.

BOX E:

Figure Nine plots the data obtained in Table 3 onto two non-logarithmic axes Z and ω. The data in this form is rather unintelligible but the regression does seem to vaguely match Z α 1/ω with a subtle positive linearity in large ω.

Taking indication from this, we invert Z and plot ω against Z^{-1} = Y. Figure Ten then reveals a very interesting pattern. The graph seems to hit a resonant peak of $(\omega_0, Y_0) = (25.3\times10^5, 4.72\times10^3)$, denoted by the vertical orange line.

The right side of the peak is decidedly linear and fits the regression in the range of data points [0,7] comfortably. Its Chi-Squared value is 5.63 ≈ 18, indicating a reasonable fit. The left-hand side has an even stronger fit in the range [11,21] (evident in the zoomed-in figure within Figure Ten) with a Chi-Squared value of $6.86\times10^{-2} < 18$. This is a surprisingly low value, but indicates a very reasonable fit.

Taking a clue from the resonant peak, we consider equations related to resonant circuits. One such example is that of a series resonant circuit involving both a capacitor and inductor, represented in equation 6a (introduction, page 2) as:

$$|Z|^2 = R^2 + (\omega L - 1/\omega C)^2$$

This already matches the pattern of Figure Nine with small ω dominated by the $-1/(\omega C)^2$ term and large ω forming a linear relationship with ωL.

Inverting Z, we get:

$$|Y|^2 = 1/(R^2 + (\omega L - 1/\omega C)^2)$$

Which similarly shows small ω increasing Y exponetially due to ωC and large ω decreasing in a 1/ω fashion.

As such, we plot tan(φ) versus ω-ω^{-1} and find that, in Figure Eleven, the pattern very roughly matches equation 6b with a Chi-Squared value of 1,000>20:

$$\tan(\varphi) = (1/R)(\omega L - 1/\omega C)$$

Small values of ω on the left-hand side of the graph hijack the slope which becomes much more linear in accordance with ωL by in higher frequencies of ω when the -1/ωC term vanishes.

Moving forward, however, we realize that the values of inductance and capacitance, L and C, are inextricably linked in the equation and necessarily difficult to isolate. We can, for example, assume that at small ω, on the left-hand side ...

Figure 9

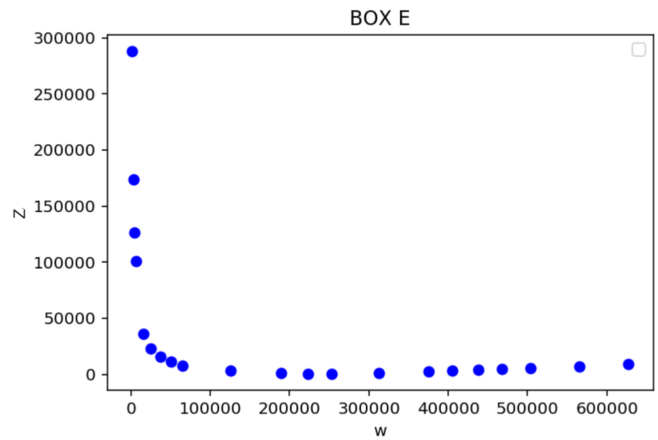
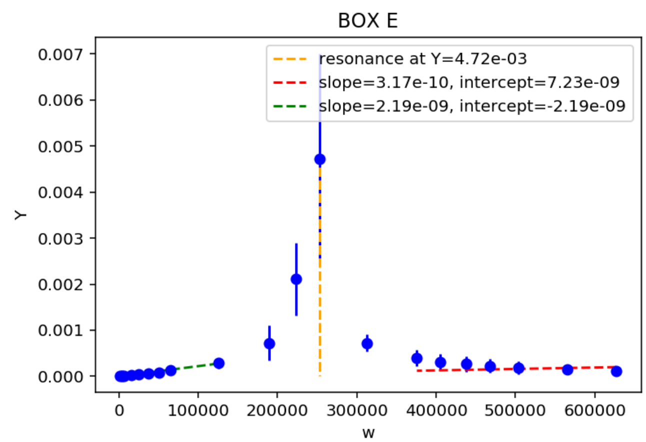
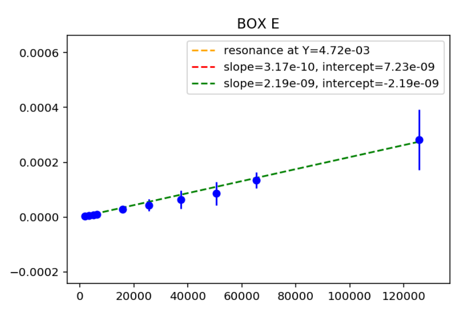


Figure 10



This plot requires two fits on either side of the peak, colored red (0 to 7) with X^2 = 5.63 ≈ 18 and colored green (11 to 21) with $X^2 = 6.86\times10^{-2}$ < 18.



This secondary figure contains a zoomed-in portion of the green regression.

... of the resonant peak in Figure 10, the term $1/\omega C$ dominates, yet ωL will still play a significant role in the actual calculable value. Despite this, there is no reliable way to calculate these individually, so we consign ourselves to calculating these with a comfortable margin of error.

The limits of where we begin to calculate each value are subjective in similar fashion to the limits of where we calculate our regressions.

As such, at large ω (data points 0 to 7), we vanish the $1/\omega C$ term:

$$|Z|^2 = R^2 + (\omega L)^2$$

And, solving for L:

$$L^2 = (Z^2 - R^2)/\omega^2$$

Calculating this term, we recieve a mean inductance L of 9.76×10^{-3} (roughly 10 millihenrys) with a standard deviation 2.99×10^{-3}.

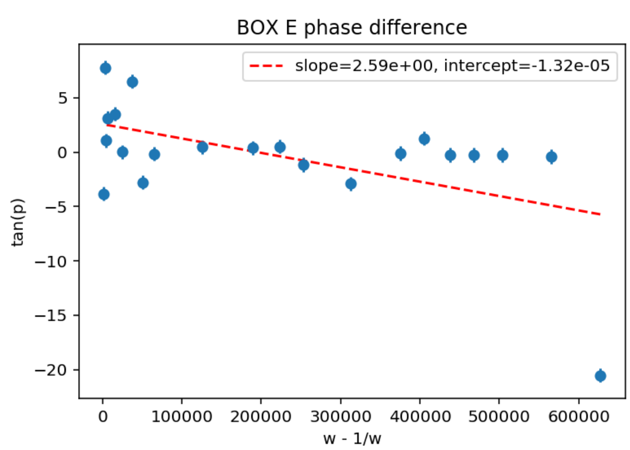
Correspondently for small ω (data points 13 to 21), we isolate $1/\omega C$ and solve for C:

$$|Z|^2 = R^2 + 1/(\omega C)^2$$

$$C^2 = 1/\omega^2(Z^2 - R^2)$$

We arrive at a mean capacitance C of 3.67×10^{-9} (roughly 4 nanofarads) with a standard deviation of 2.89×10^{-9}.

Figure 11



This fit significantly disagrees with its given data, which is self-evident just looking at the spread of data. Interestingly, this 'incorrect' negative skew seems partially the result of a single separate point on the lower-right.

X^2 = 1,000 >> 20

DISCUSSIONS AND CONCLUSIONS:

This last major section is dedicated to various comments needed on the experiment and, largely, to our mistakes: where we went wrong. I will discuss possible remedies to our mistakes and suggestions for future experiments with similar goals.

First, likely the most important mistake, was our error-taking and analysis methods. We unknowingly recorded our data without tracking uncertainty for every data point and, as such, needed to make larger-scale assumptions as to the error in each parameter.

This causes an issue: overestimating error will weaken the integrity of the goodness-of-fit test; underestimating it would misrepresent our data and potentially mislead us. I believe that the values we chose reflect the variance I observed while taking the data and are adequate to estimate the error for our purposes.

Frequency was both dialed in and measured with the oscilloscope and was recorded with an accuracy down to 1/1000th. For this reason, we decided to consider its error negligible. Resistance is a little less certain, however with testing different values with an ohmmeter across the resistance box, an error of about 5Ω seems reasonable. The voltage across the resistance and mystery boxes each tended to vary in the oscilloscope by no more than 1/10th of a volt. We decided to make this our error. Lastly, phase tended to vary the most out of all our values; we settled on about 10 which represented the majority of those variations.

Again, the ideal situation would have been to immediately record these variances as we took in our data points. Our assumptions, however, are not baseless. I would consider these assumptions of error to be reasonabe.

Transitioning now into issues arising from the treatment of individual boxes, one naturally arising question is that of the two sets of data recorded of Box B. Does disregarding some part of the data constitute deleting it? Does sharing data between the sets constitute modification? The context shines quite a bit of light on the situation. We had actually recorded Box B for 20 data points before, but made an error in the actual set-up of our circuit. For that reason, we had already built a program to plot the points and calculate a regression. As we plugged our 'first set' (Table 1a) of data points into the program, we immediately noticed the pattern and decided to keep the data but start an alternative set at the point where we began changing resistance. Our set-up between the two sets remained unchanged; both sets were recorded in the same session and, lastly, both sets are used to make different arguments that reach the same conclusion in our report.

One essential recommendation I would make for future experimental attempts would be to record more consistent coverage across the entire range of frequencies. As I mentioned briefly in the analysis of Box B, our measurements tended to be very dense at low frequencies and sparser at higher frequencies. This is because our signal generator increased by multiplicative factors (e.g. 10, 100, 1000, ...) and so smaller changes at large frequencies were more difficult to dial in accurately.

One last points which covers all the boxes is our issues with apparent outliers appearing in our graphs of tan(φ) vs. ω. Quite commonly, points appear far outside the general cluster. As the domain of the tan function is asymptotic at all 3πn/2 as n∈Z, the sensitivity of values ω at near these points is higher. I attribute these 'outliers' to points of phase which we recorded innaccurately, and too close to these asymptotes. I believe a more rigorous recording of the uncertainty of the phase from the outset would have mitigated the effect these points had on the data.

ACKNOWLEDGEMENTS AND SOURCES:

This report was created with data performed in an experiment done together with Scarlet Passer.

I received an immense amount of help from both the teaching assistant, Drew Bischel, and Professor of Physics 133, Aiming Yan.

I appreciate the your tireless patience with an experiment that required many wrong turns to make right.

The following texts were used in this report. They are referenced throughout the report in MLA citation format: (Author, Page Number).

1) George Brown et. al., Physical Sciences Department. "Manual for PHYS 133-01". Lab handbook. University of California, Santa Cruz. 2019. Print.

2) Aiming Yan, "Physics 133 Statistics Lecture 3" . Accessed October 30, 2019

3) Wikipedia contributors, "RC circuit," Wikipedia, The Free Encyclopedia, https://en.wikipedia.org/w/index.php?title=RC_circuit&oldid=921663113 (accessed October 30, 2019).

TABLES:

Box B: Table 1a (first set)

f (Hz)	R (Ω)	ΔV_M (V)	ΔV_R (V)
51.2	*50*	*2.96*	*2.80*
101	*50*	*3.00*	*2.80*
151	*50*	*3.00*	*2.76*
202	*50*	*3.00*	*2.76*
304	*50*	*3.04*	*2.76*
399	*50*	*3.08*	*2.72*
502	*50*	*3.16*	*2.72*
654	*50*	*3.24*	*2.68*
901	*50*	*3.40*	*2.56*
1.02×10^3	50	3.52	2.52
1.52×10^3	50	3.88	2.36
2.03×10^3	100	3.28	3.20
2.50×10^3	100	3.60	3.04

Box B: Table 1a continued

f (Hz)	R (Ω)	ΔV_M (V)	ΔV_R (V)
3.00×10^3	100	3.96	2.84
4.00×10^3	100	4.52	2.60
5.00×10^3	150	4.32	3.00
6.00×10^3	150	4.60	2.76
7.00×10^3	150	4.84	2.48
8.00×10^3	150	5.00	2.28
9.00×10^3	150	5.16	2.12
1.00×10^4	500	3.64	4.28
1.50×10^4	500	4.44	3.60
2.00×10^4	500	4.96	2.96
2.50×10^4	500	5.24	2.252
3.00×10^4	500	5.36	2.20
4.00×10^4	1500	4.64	3.88
4.50×10^4	1500	4.84	3.64
5.00×10^4	1500	5.00	3.40
6.00×10^4	1500	5.28	3.00
7.00×10^4	1500	5.48	2.56
7.50×10^4	3500	4.08	4.68
8.00×10^4	3500	5.32	4.48
8.50×10^4	3500	5.48	4.32
9.00×10^4	3500	5.64	4.08
9.27×10^4	3500	5.72	4.04
9.50×10^4	3500	5.80	3.92
9.75×10^3	3500	5.92	3.89
1.00×10^5	3500	5.96	3.64

Box B: Table 1b (second set)

f (Hz)	R (Ω)	ΔV_M (V)	ΔV_R (V)	φ
51.2	*50*	*2.96*	*2.80*	*1.47*
101	*50*	*3.00*	*2.80*	*4.25*
151	*50*	*3.00*	*2.76*	*7.59*
202	*50*	*3.00*	*2.76*	*9.45*
304	*50*	*3.04*	*2.76*	*13.1*
399	*50*	*3.08*	*2.72*	*17.3*
502	*50*	*3.16*	*2.72*	*22.7*
654	*50*	*3.24*	*2.68*	*27.1*
901	*50*	*3.40*	*2.56*	*34.1*
1.02×10^3	50	3.52	2.52	38.1
1.52×10^3	50	3.88	2.36	48.3
1.99×10^3	50	4.24	2.12	54.0
2.24×10^3	50	4.40	2.04	56.1
2.49×10^3	50	4.56	1.96	58.1
2.75×10^3	50	4.64	1.84	60.3
3.00×10^3	50	4.72	1.80	62.9
3.50×10^3	50	4.88	1.60	65.7
4.00×10^3	50	5.04	1.48	67.0
4.50×10^3	50	5.16	1.36	69.4
5.00×10^3	50	5.24	1.24	70.6

Box D: Table 2

f (Hz)	R (Ω)	ΔV_M (V)	ΔV_R (V)	φ
50.0	4.00×10^4	2.04	3.04	88.0
99.5	4.00×10^4	2.88	2.32	89.3
152	4.00×10^4	3.28	1.76	91.1
299	7.00×10^4	2.12	2.96	91.3
497	7.00×10^4	2.76	2.36	88.7
799	7.00×10^4	3.16	1.76	88.0
920	3.00×10^4	2.40	2.64	87.7
1.55×10^3	3.00×10^4	2.96	1.96	85.2
2.00×10^3	3.00×10^4	3.16	1.64	84.3
2.47×10^3	1.00×10^4	2.20	2.72	84.7
2.94×10^3	1.00×10^4	2.44	2.52	84.5
3.98×10^3	1.00×10^4	2.72	2.12	80.6
4.95×10^3	1.00×10^4	2.84	1.84	79.8
5.99×10^3	400	2.12	2.76	79.6
7.52×10^3	400	2.36	2.44	76.8
1.03×10^4	400	2.56	2.08	67.3
1.43×10^4	400	2.76	1.68	64.3
1.91×10^4	100	1.48	2.76	56.6
2.46×10^4	100	1.64	2.56	55.0
3.00×10^4	100	1.72	2.44	45.1
4.99×10^4	100	1.76	2.12	34.0
7.02×10^4	100	1.80	2.00	22.9
1.00×10^5	100	1.80	1.92	16.9

Box E: Table 3

f (Hz)	R (Ω)	ΔV_M (V)	ΔV_R (V)	φ
99.8×10^3	6.00×10^3	4.24	6.24	121
89.9×10^3	6.00×10^3	5.50	5.76	119
80.1×10^3	6.00×10^3	5.56	4.88	116
74.5×10^3	6.00×10^3	5.72	4.28	116
69.7×10^3	6.00×10^3	5.84	3.68	116
64.4×10^3	6.00×10^3	5.84	3.08	114
59.8×10^3	6.00×10^3	5.80	2.48	113
49.8×10^3	6.00×10^3	5.72	1.32	115
40.3×10^3	200	3.36	3.56	71.4
35.5×10^3	200	2.06	4.84	85.3
30.1×10^3	1.50×10^3	3.92	3.64	85.2
20.0×10^3	1.50×10^3	2.12	5.00	88.4
10.4×10^3	1.50×10^3	1.08	5.36	94.1
8.05×10^3	15.0×10^3	4.12	3.20	83.6
5.94×10^3	15.0×10^3	3.64	3.80	83.1
4.06×10^3	15.0×10^3	2.92	4.44	88.0
2.52×10^3	15.0×10^3	2.06	4.96	92.4
1.01×10^3	1.00×10^5	3.60	3.64	79.8
800	1.00×10^5	3.16	4.00	82.5
548	1.00×10^5	2.56	4.44	73.7
305	1.00×10^5	1.68	4.84	67.8