An Analysis of the Atomic Spectra of Helium, Hydrogen and Neon

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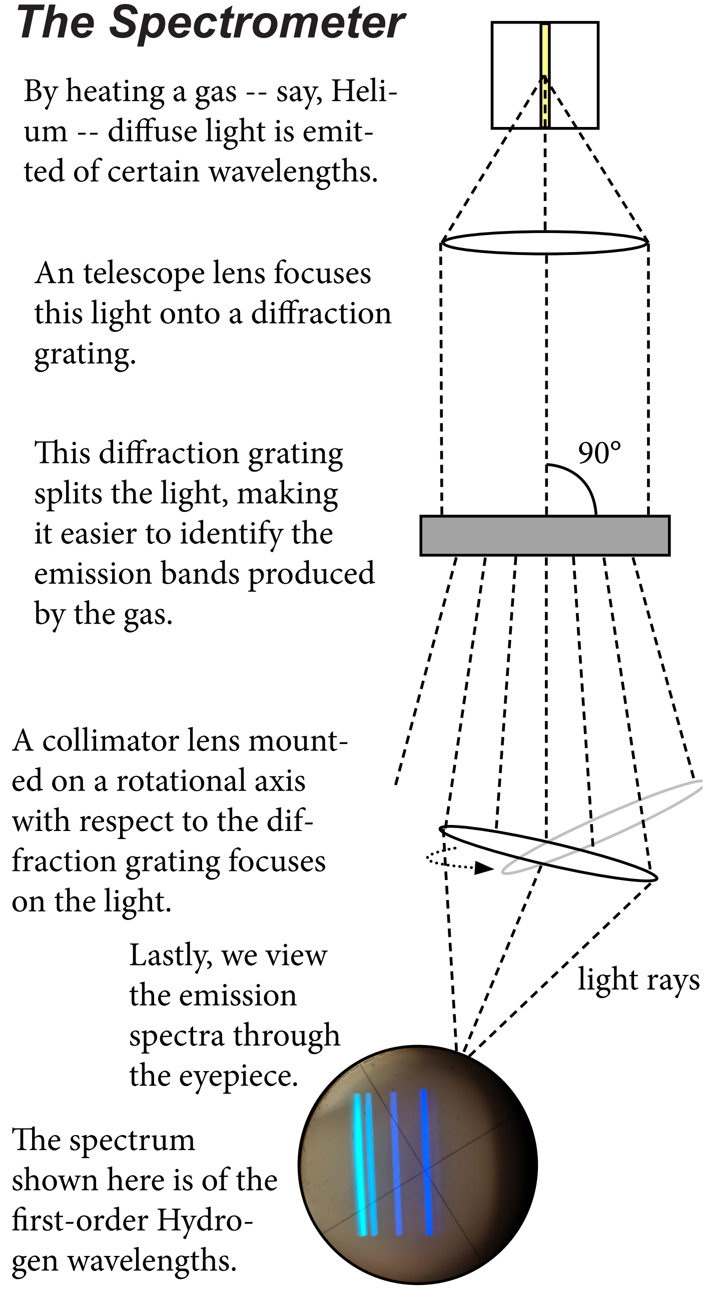
ABSTRACT

This report explores the nature of the diffraction of light, and the absorption or emission of different wavelengths of light as a result of the atoms involved.

We analyze atomic emission spectra of Helium, Hydrogen and two different combinations of Neon-Hydrogen gas through a diffraction grating and match them to accepted experimental wavelengths. Subsequently, we attempt to determine a weighted diffraction grating constant d_w = 3.286 ± 8.339 nanometers between lines or, 304.3 ± 119.9 lines per millimeter; an value which falls within our factory-rated 310 lines per millimeter but reflects an inherent problem in the error propagation across the entirety of the data set. We attempt to determine a weighted Rhydberg constant for Hydrogen, R_W = 11349512 ± 0.034280326 m^{-1} which does not fit our accepted experimental value R_H = 10967758 m^{-1} despite a fractional difference of just 3.48 percent.

As such, though the goals of the experiment are met, the resultant experimental values are inconclusive and further analysis is recommended to reconcile these problems.

The Spectrometer



INTRODUCTION

The rainbow is as ubiquitous to human experience as it is mysterious. Tiny particles of water, suspended in Earth's atmosphere, diffract the Sun's colored guts across the sky in dazzling fashion. This same mechanism is true in the many other forms of rainbows we find in nature, including that which Isaac Newton found in 17th century when he aimed light through a prism[1].

It took another century and a half before physicists discovered dark lines -- shadows -- in their spectra. Shadows which the constituent elements themselves produced.

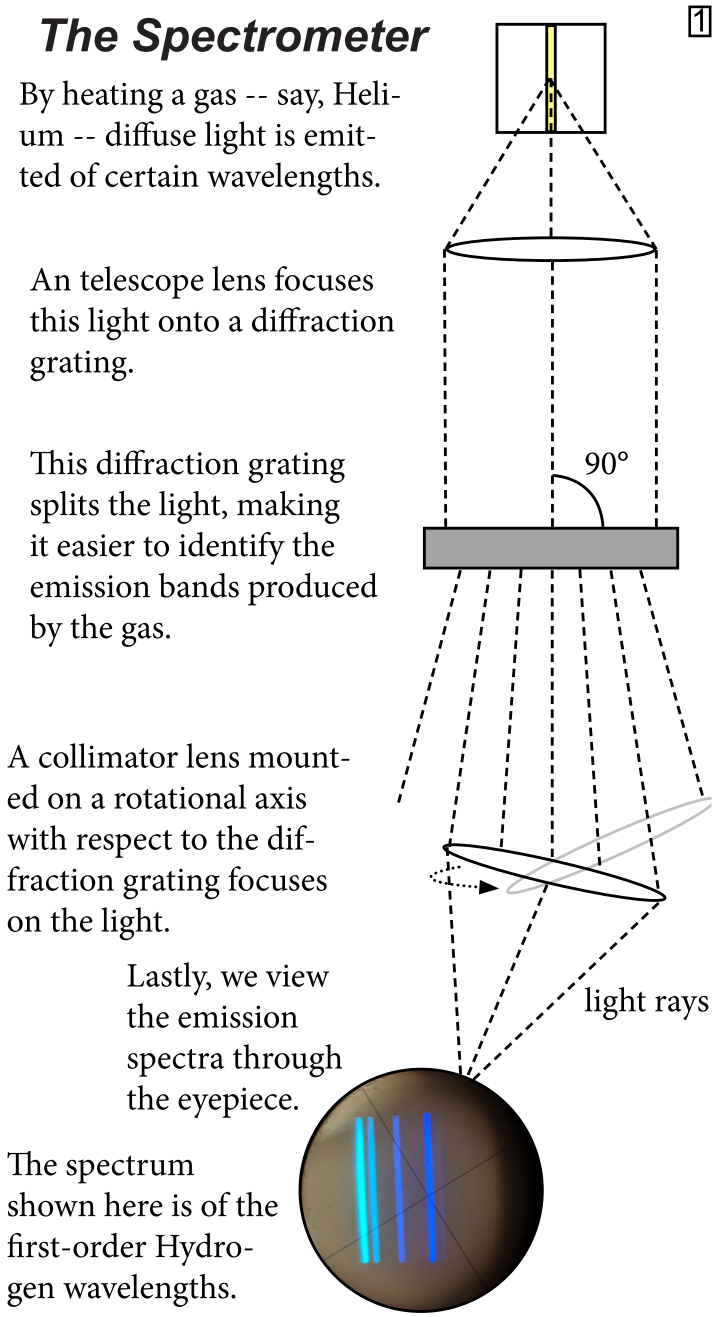
Emission spectroscopy analyzes these lines, raising the energy of different gases and compounds until they emit light. Their electrons, locked into different circuits in a quantum race-track, jump suddenly onto different paths, releasing a burst of energy. This energy comes in the form of light at different intensities and frequencies. Analyzing these patterns of light, as in that of shown at the end of Figure One, allows us to determine the spectrum to which a source of light belongs.

Our goal in this experiment is to use a spectrometer to analyze the emission spectra of Helium, Hydrogen and Neon-based light and empherically determine the spacing of our diffraction grating and the associated Rhydberg constant for Hydrogen.

The following section, APPARATUS AND PROCEDURE, describes in detail how our experiment was conducted. Raw data from the experiment is collected in the TABLES section appended to this report.

I will subsequently analyze our raw data in RESULTS, including error analysis, and make some experimental measurements and conclusions. Lastly, DISCUSSIONS AND CONCLUSIONS comments on shortcomings of the experiment and suggests improvements for subsequent iterations of this experiment. The report is appended by ACKNOWLEDGEMENTS AND SOURCES and the TABLES section.

Figure 1



This graphic breaks down the constituent components of the spectrometer and describes their functions and intended effect on the light rays emitted by a gas lamp.

[1] Wikipedia, History of Spectrosopy
[2] Brown, Lab Manual

APPARATUS AND PROCEDURE:

The most essential factor when using a spectrometer is making sure that it is properly aligned.

Alignment first starts by focusing the collimator objective lens on the light-producing slit. The diffraction grating, which sits on a table, can be replaced by a mirror by which we can finely adjust the leveling and rotation of the table so our eyepiece is properly level. Lastly, we can change the measurement scale (in our case, a Vernier scale) to a convenient place so we can measure the difference in angles.

This difference in angles is how we ensure that our spectrosope is properly aligned. We can set a threshold error value (in our case, 10 arcminutes) which we do not want to exceed. Our error value for each angle is thus[2]:

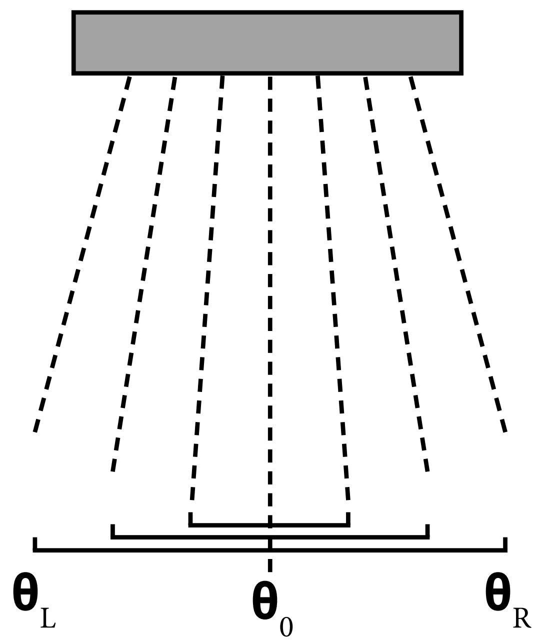
$$\delta\theta_i = |\theta_R - \theta_0| - |\theta_L - \theta_0| < 10 \text{ arcminutes}$$

Note, however, that our recorded angles for the raw data in the TABLES section are already the adjusted values $\theta_R - \theta_0$ and $\theta_L - \theta_0$ in order to make the data more understandable.

For each angle, we record its color and order, or how many times that color has already appeared.

For each experiment, the data collected varies slightly according to their needs. For Helium, we recorded color, order, angle and include the known wavelengths for each. For Hydrogen, we include additionally the computed wavelenghts, which I discuss more in the RESULTS section. Our comparison between Helium-Neon gas and a Neon laser uses merely four data points which comparitively analyze the light emitted by Neon.

Figure 2 [1]



This figure depicts light rays passing through the diffraction grating. The central ray perpendicular to the grating is the zero-angle θ_0. Angles counterclockwise to θ_0 are defined as θ_R wjereas those clockwise to θ_0 are θ_L. These definitions are relevant to the TABLES section.

The error for each pair of angles $\delta\theta_i$ we calculate by finding the difference from the zero point θ_0 for each pair θ_L and θ_R and finding the difference of the absolute value between them: $|\theta_R - \theta_0| - |\theta_L - \theta_0|$. We ensure that for each pair, their error does not exceed 10 arcminutes.

[1] Brown, Lab Manual

RESULTS

Helium

The goal of our analysis of the Helium spectrum is to use it as a wavelenght standard to measure the spacing of lines on our diffraction grating. The wavelengths we assign to each color we determine using the known data shown in Figure Three.

Table One collates both our data on color, order and angle and includes the known wavelength data. Figure Four shows this data graphed, with the theoretical wavelengths compared to the angle θ.

This data we can then manipulate using[2]:

$$m\lambda/d = \sin\theta \quad \text{eq. 1}$$

wherein for each data point i:

$$d_i = m_i\lambda_i/\sin(\theta_i)$$

Obviously, our calculation should include error, so we need to include our propagation of the error in this function[3]:

eq. set 2: ***for some q = q(x,y)***

$$\sigma_q = \delta q = \sqrt{[(\partial q/\partial x \times \sigma_x)^2 + (\partial q/\partial y \times \sigma_y)^2]}$$

in our case,

$$\delta d_i = (d/d\theta_i) \times \sigma_e$$
$$= -m_i \times \sigma_e \times \lambda_i \times \cos(\theta_i) / \sin^2(\theta_i)$$

With this information, we can then calculate the weighted average d_w and its error σ_d using[3]:

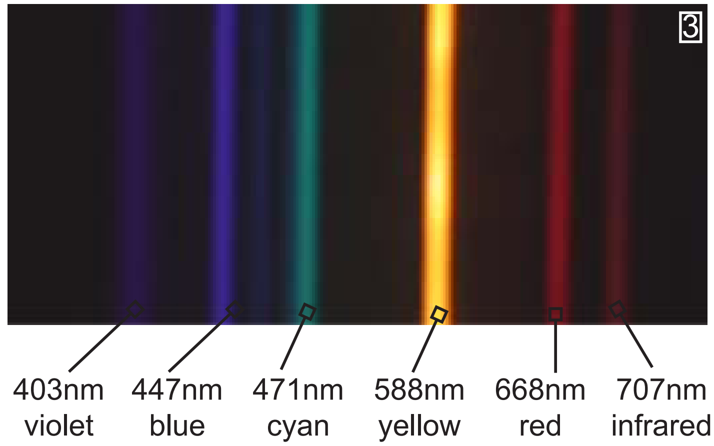
eq. set 3: ***for some parameter x***

$$x_w = \Sigma w_i x_i / \Sigma w_i$$
$$\sigma_x = (\Sigma w_i)^{-1/2}$$
$$\text{where } w_i = (\delta x_i)^{-2}$$

We then are able to determine d_w = 3.286± 8.339 nanometers between lines which, when inverted, gives us 304.3 ± 119.9 lines per millimeter.

The standout feature in this calculation is the uncertainty in d_w, which will be discussed further as a shortcoming of our experiment in DISCUSSIONS AND CONCLUSIONS. It is worth noting, however, that our value is rather close to our factory listed 310 lines per millimeter, and is therefore an acceptable result for our purposes going forward.

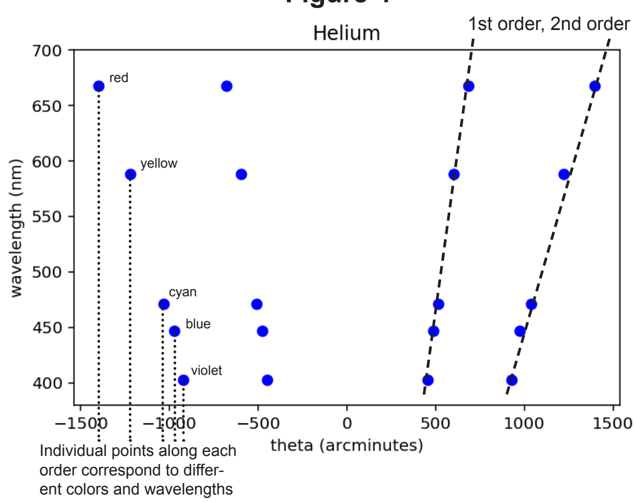
Figure 3



This composite image shows the emission spectrum of Helium and its associated known wavelengths.

The wavelength data shown here is obtained from page 67 of the lab manual[1].

Figure 4



Plotting the data obtained in Table One -- expected wavelength vs. the angle from θ_0.

Keep note that the visible spectrum is between 400 and 700 nm. There are wavelengths emitted outside this range, however we are not able to observe them in this fashion.

[2] Brown, Lab Manual, [3]Hyperphysics, Atomic Spectra
[4]Yan, Statistics Lecture 3

Hydrogen

Determining a constant for our diffraction grating proves particularly useful going forward so that we can produce a wavelength for our data.

The goal of our Hydrogen analysis is to determine its Rhydberg constant (R_H), which directly connects the atoms' change in energy state with correspondent wavelengths we observe as the emission spectrum:

$$1/\lambda = R_H\,(n_f^{-2} - n_i^{-2})$$

where n_i represents the initial energy state and n_f represents the final state.

All of the wavelengths that lie in the visible spectrum belong to the Balmer series, so named after the schoolteacher who discovered them. Common among them is that their final energy states, n_f, are always the second (see Figure Five).

As such, we can make use of the equation:

$$1/\lambda = R_H\,(0.25 - n_i^{-2}) \qquad \text{eq. 4}$$

$$n_i = 3, 4, 5, 6$$

for all of the visible-spectrum wavelengths we will be dealing with in this report.

Figure Five shows all the visible-spectrum colors -- of which, ultraviolet was too faint to accurately measure. In fact, even violet was too faint to be recorded in the second order. As such, we recorded five data points over just three colors: violet, blue and red.

We can find theoretical wavelengths using a known 'accepted' Rhydberg constant[2] R_H = 10967758 m^{-1}. These are listed in the figure as 434, 486 and 656 nm for changes to E_2 from energy levels 5, 4 and 3 respectively.

Table Two, however, includes experimental wavelengths computed from a modified equation one:

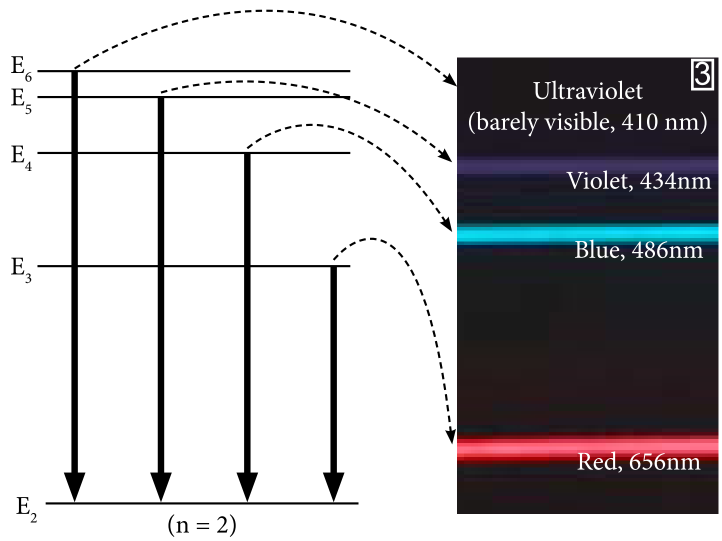
$$\lambda_{exp} = d_w \times \sin(\theta_i) / m_i \qquad \text{eq. 5}$$

in which we get d_w from the Helium experiment.

Figure Six plots both the theoretical and experimental wavelengths against the angle. As is evident, there seems to be a downards systemic shift in the experimental wavelengths. This is possibly due to the large error in d_w, given that the shift seems to be consistent in magnitude across each set wavelength (and $\lambda \sim d_w$).

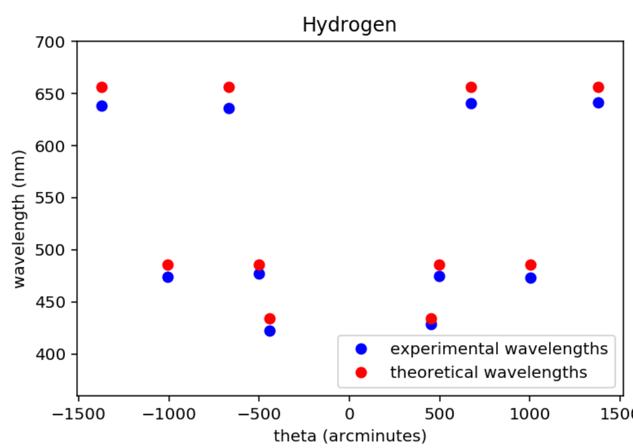
Figure 5

The Balmer Series (visible Hydrogen spectrum)



The wavelengths shown are computed using the equation $1/\lambda = R_H\,(0.25 - n_i^{-2})$ wherein n_i = 3, 4, 5, 6 and R_H = 0967758 m^{-1}.

Figure 6



Plotting the data obtained in Table Two -- computed wavelength vs. the angle from θ_0.

Both the experimental and theoretical wavelengths are plotted against the angle θ, making apparent the systemic error present in the data.

[2] Brown, Lab Manual, [3]Hyperphysics, Atomic Spectra
[4]Yan, Statistics Lecture 3

Moving forward, however, we can match each of these wavelengths to their respective energy states n_i= 3,4,5 and calculate a weighted experimental value for the Rhydberg constant ourselves using equation four:

$$R_i = 1/(\lambda_i(0.25 - n_i^{-2}))$$

and propagating error using equation set two:

$$\delta\lambda_i = \sqrt{(d\lambda/d\theta \times \delta\theta_i)^2 + (d\lambda/dd_w \times \sigma_d)^2}$$
$$\text{where } d\lambda/d\theta = (d_w/m_i) \times \cos\theta_i$$
$$\text{and } d\lambda/dd_w = \sin\theta_i / m_i$$

$$\delta R_i = (d/d\lambda_i) \times \delta\lambda_i$$
$$= -1 \times \lambda^{-2} \times 1/(1/4-n\hat{}-2) \times \delta\lambda_i$$

wherein our weighted value and error are:

$$R_w = \Sigma w_i R_i / \Sigma w_i$$
$$\sigma_R = (\Sigma w_i)^{-1/2}$$
$$\text{where } w_i = (\delta R_i)^{-2}$$

from equation set three.

Computing these equations, we arrive at a value R_W = 11349512 ± 0.034280326 m^{-1}. The shortcoming in this data is again the error, which presents itself as a very, very small value. This is touched on in the subsequent section but is likely the result of the large error we found when calculating the weighted value for the diffraction grating. Even so, our value is quite close to our accepted value 10967758 m^{-1}. In fact, their fractional difference $(R_w-R_H)/R_H$ is just 3.48 percent!

Neon

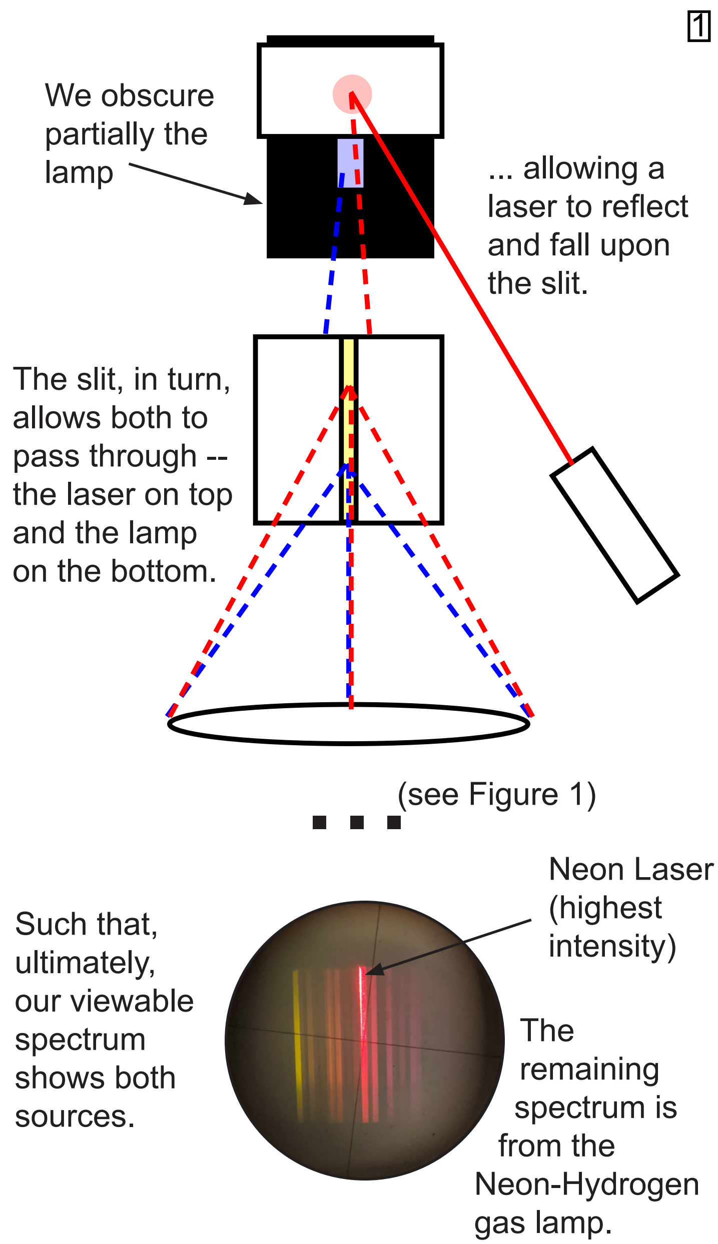
The third and final experiment compares the emission spectra of a Helium-Hydrogen gas to that of a Neon laser.

In order to perform the experiment, a slight change was made to the the setup to show both spectra simultaneously, shown in Figure Seven.

Covering the top of the Helium-Neon lamp, we can aim the laser so that its light reflects and enters the slit in tandem with the lamp's. The laser's light enters the telescope lens at the top while the lamp's enters at the bottom. Passing through the entirety of our spectroscope, the resulting image is a combination of the two, with the gas lamp's diffuse light standing as a template while the Neon laser overlays intensely at its sole emission wavelength.

This is exactly what we would expect. The lamp shines at wavelengths of both Hydrogen and Neon -- not some combination -- as the atoms operate independently from one another when absorbing and remmitting energy. The Neon laser shines at the precise wavelength that the Neon in the gas does, and so the two overlap perfectly.

Figure 7



This graphic explains how we block part of the Hydrogen-Helium lamp to allow the laser to reflect, thereby permitting both light sources to enter the spectroscope.

Table Three records the angle across two orders of the Neon laser and includes the computations for its wavelength via equation five.

It's worth noting that, while the Neon in the lamp shines at other wavelengths, the laser shines only at a wavelength of about 632.8 nm[2], a statistical trick achieved by varying pressures of Neon and Helium within the tube to stimulate a specific emission.

[1] Brown, Lab Manual

Figure Eight plots the data from Table Three, showing the small deviation among the points (just 2.1 nm) but a significantly different average, 616.7 nm, from our expected value of 632.8 nm.

There are several contributing factors to this error. The first is the high quantity of wavelengths emitted in the red area of the visible spectrum. Figure Nine shows the large number of these red emission lines, which could be confused and possibly introduce random error. As the standard deviation of this set is rather low, however, we can conclude that this is not likely.

Rather, the skewed d_w value, as is the case in our other two experiments, is the likely culprit. This is discussed in much more detail in the subsequent section.

DISCUSSIONS AND CONCLUSIONS

The most glaring problem in this report is that of the large computational error for d_w. This problem is shared with the propagation for wavelength. The problem arises when, using equation set two, one takes the derivative with respect to theta:

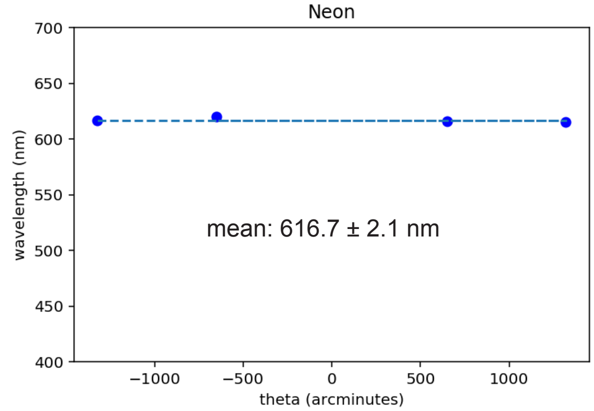
$$d/d\theta_i = -m_i \times \bar{\lambda}_i \times \cos(\theta_i) / \sin^2(\theta_i)$$

The specific element of the equation we want to focus on is the value of $\cos(\theta)/\sin^2(\theta)$. When we run the values for theta, we convert our value for theta from arcminutes to radians. Since the problem lies with the error, we can take a value of 10 arcminutes as an example. For each radian, there are 3437.75 minutes of arc. We can divide our number of arcminutes by this value to obtain roughly 0.029 radians. Keep in mind that all values of error we use will be of this value or smaller.

The problem arises when we take the sin(0.029 radians) -- essentially zero. And this is the maximum value it can obtain given our imposed constraints for error. The cosine of the same number comes out to about 1 (its minimum). As is likely immediately evident, $1/(\text{nearly } 0)^2$ is a very small number -- in our case, roughly 118,000.

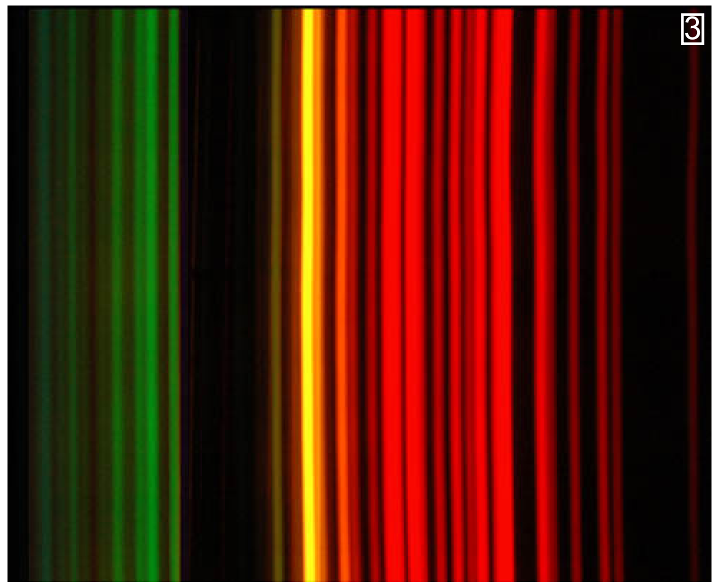
Therein lies our problem -- when propagating error for d_w, and when using this value in subsequent experiments, we risk blowing up our error with these high quantities. When propagating error for $\bar{\lambda}$, such as the case when calculating for the Rhydberg constant, we run into the same problem.

Figure 8



Similar to Figures Four and Six, this figure graphs the singular bright red wavelength across two orders. Additional wavelengths (and colors) are considered extraneous to this experiment but are depicted below, in Figure Nine.

Figure 9



This is a composite image of the Neon spectrum. Note the high density of wavelengths in red and orange.

As we've seen, the error in each individual $\delta\bar{\lambda}_i$ is quite large -- and when we calculate the error for the Rhydberg constant, we determine it with:

$$\sigma_R = (\Sigma(\delta R_i^{-2}))^{-1/2}$$

$$\text{where } \delta R_i = -\bar{\lambda}^{-2} \times 1/(1/4 - n\text{^}{-2}) \times \delta\bar{\lambda}_i$$

wherein we can see that δR_i are very large but the error σ_R consequently becomes small.

The discussion going forward in relation to this data is exploring instead a different, more consistent approach for the propagation of error. Where necessary, the data itself varies little -- such as the standard deviation in the computed wavelengths of the Neon laser. In spite of the systemic error, the values are remarkably precise.

[3] Hyperphysics, Atomic Spectra

Further experimentation would benefit different statistical analyses of the the data and a larger, repeated sample pool so as to determine inherent errors in the data.

Another source of error is the calibration of the spectroscope, which may need to be recalibrated more often; though this would likely present itself instead in the form of random error.

Altogether, the goals of this experiment were to analyze the emission spectra of four different gases, determine a constant for the diffraction grating and use it to determine a Rhydberg constant for Hydrogen.

All these goals were achieved to varying levels of success. The emission spectra of Helium, Hydrogen and two combinations of Neon-Hydrogen gas were analyzed and codified in accordance with accepted experimental data.

We were able to determine a diffraction grating value $d_w = 3.286 \pm 8.339$ nanometers between lines or, 304.3 ± 119.9 lines per millimeter -- most definitely within the bounds for our factory rating of 310 lines per millimeter yet evidence of an inherent problem in error propagation.

Similarly with the Rhydberg constant for Hydrogen, we found $R_W = 11349512 \pm 0.034280326\ m^{-1}$ which does not fit our accepted experimental value $R_H = 10967758\ m^{-1}$ despite a fractional difference of just 3.48 percent.

As such, though this report's goals were met, its experimental results are inconclusive. Further testing and data analysis are needed to reconcile these problems.

ACKNOWLEDGEMENTS AND SOURCES:

This report was created with data performed in an experiment done together with Steven Soares.

I received an immense amount of help from both the teaching assistant, Drew Bischel, and Professor of Physics 133, Aiming Yan.

I appreciate the your tireless patience with a learning experience that requires many wrong turns to make right.

This lab report marks the last in the course, and stands as the culmination of a quarter and the knowledge I've gained throughout it. Perhaps the most important lesson I've been taught is that it's better to know when you are wrong, than to pretend you are right.

Thank you.

The following texts were used in this report. They are referenced throughout the report in MLA citation format: (Author, Page Number).

1) Wikipedia contributors, "History of spectroscopy," Wikipedia, The Free Encyclopedia, https://en.wikipedia.org/w/index.php?title=History_of_spectroscopy&oldid=919740254 (accessed December 5, 2019).

2) George Brown et. al., Physical Sciences Department. “Manual for PHYS 133-01”. Lab handbook. University of California, Santa Cruz. 2019. Print.

3) HyperPhysics, “Atomic Spectra”, http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/atspect.html, http://hyperphysics.phy-astr.gsu.edu/hbase/hyde.html#c4, http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/atspect2.html#c1 (accessed December 5, 2019)

TABLES

Table 1
The Helium Spectrum

Color, Order	Angle $(\theta-\theta_9)$ (arcminutes)	Expected[1] Wavelength (10^{-9} m)	Error (arcmin-utes)
Violet, 1	-450	403	4
Blue, 1	-480	447	5
Cyan, 1	-510	471	2
Yellow, 1	-599	588	3
Red, 1	-682	668	2
Infr., 1	-715	720	5
Violet, 1	456	403	4
Blue, 1	485	447	5
Cyan, 1	512	471	2
Yellow, 1	602	588	3
Red, 1	684	668	2
Infr., 1	720	720	5
Violet, 2	-925	403	0
Blue, 2	-973	447	6
Cyan, 2	-1033	471	3
Yellow, 2	-1222	588	0
Red, 2	-1399	668	3
Violet, 2	925	403	0
Blue, 2	975	447	2
Cyan, 2	1039	471	6
Yellow, 2	1222	588	0
Red, 2	1396	668	3

A note on the tables: the angle is represented in arcminutes via the equation $(\theta-\theta_0)$, which makes angles clockwise of θ_0 negative, and angles counter-clockwise positive.

Table 2
The Hydrogen Spectrum

Color, Order	Angle $(\theta-\theta_9)$ (arcminutes)	Calculated Wavelength (10^{-9} m)	Error (arcmin-utes)
Violet, 1	450	428.9	7
Blue, 1	499	475.3	2
Red, 1	675	641.0	5
Violet, 1	-443	422.3	7
Blue, 1	-501	477.2	2
Red, 1	-670	636.3	5
Blue, 2	1005	473.5	1
Red, 2	1379	641.5	8
Blue, 2	-1006	473.9	1
Red, 2	-1371	638.0	8

Where, for the Hydrogen spectrum, the expected wavelengths are:

Violet = 343 nm
Blue = 486 nm
Red = 656 nm

Table 3
The Red Neon Laser

Color, Order	Angle $(\theta-\theta_9)$ (arcminutes)	Calculated Wavelength (10^{-9} m)	Error (arcmin-utes)
Red, 1	648	615.7	5
Red, 1	-653	620.4	5
Red, 2	1319	615.0	3
Red, 2	-1322	616.9	3

Where the expected wavelength is 632.8 nm.

[1] Brown, Lab Manual