First-order filters for analog signals

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Abstract:

This report explores three first-order filters: the low- and high-pass passive resistor-capacitor circuits, and the active low-pass operational-amplifier circuit.

With an emphasis on analog signals, we discuss the effects of these filters including attenuation or amplification of input voltage relative to output voltage and the phase differences between input and output signals.

With this information, we create logarithmic bode plots and phase plots relative to frequency for each of the filters. The pattern, steepness and polarity of these relationships are discussed, with emphasis on why they change between one another.

The concept of a corner frequency -- at which the phase is exactly 45 degrees -- is discussed and confirmed in the context of our three circuits.

Introduction:

An analog signal follows the patterns described by the trigonometric functions of sine and cosine. The difference between the two, however, is a simple difference in their phase ϕ; the delay between their peaks. Cosine can be represented by sine simply as $\cos(t + \pi/2) = \sin(t)$.

Differences in phase are the subject of this report. In electronic circuits, we can create low- and high-pass filters to lag and amplify output signals, with different circuits conveying different relationships between the phase difference and input frequency.

This report explores three first-order circuits: the low- and high-pass RC filters and an op-amp low-pass filter. Our goals with each will be to evaluate their corner frequency f_c and find the phase shift at this frequency. Measuring phase in a range of frequencies around f_c helps us visualize the lagging of the output signal. We can similarly collect data for the frequency relative to the logarithmic magnitude of the output relative to the input signal to create a bode plot.

Methods:

The goals of this experiment are rather straightforward. As such, all that requires explanation are the circuit design and the mathematical principles that dictate changes to the output signal.

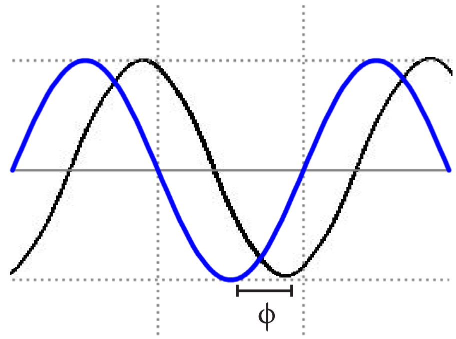
Figure Two lays out each of the filters' schematic design, from the RC design, consituting simply a resistor and capacitor, to the op-amp design, which includes an operational amplifier. Hooking up an oscilloscope, we measure the voltage signal across the signal generator (+ -) and compare it to the output signal V_{out}.

When taking data for the phase difference, we can simply compare the waveforms from the input and output signals. Taking their maximum voltages allows us to create the bode plot, which is be described later.

The corner frequency $f_c = 1/(2\pi RC)$ describes the frequency at which the phase difference will theoretically be exactly 45 degrees, where R and C are the resistance and capacitance of the circuit (R_f and C_f in the case of the op-amp circuit). By plotting the changing frequency versus the phase difference, we should be able to see this relationship around the corner frequency naturally. This should thereby give us a sense of the range of frequencies we need to explore.

When creating the bode plot, we take the ratio of the input and output voltages and convert it to the logarithmic decibel (dB) scale. Specifically, we define the magnitude $M = 20 \times \log_{10}(V_{out}/V_{in})$.

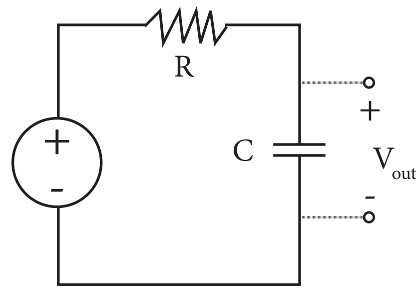
Figure One: Difference in analog phase



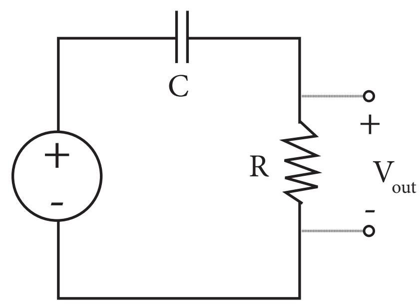
This simple diagram illustrates two analog signals and the phase difference between them. All analog signals will follow the sine function, with the cosine function simply the same function with a phase difference of $+\pi/2$.

Figure Two: Circuit schema of Low- and High-pass filters

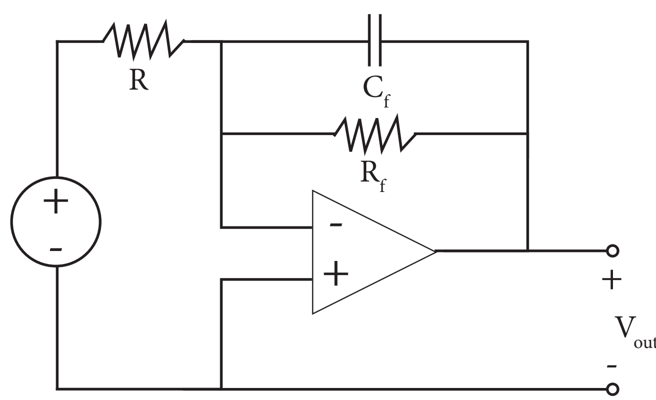
(a) Low-pass RC filter



(b) High-pass RC filter



(c) Low-pass operational amplifier filter



These are the circuits which we will be testing in this report. It is worth particular note that the values R and C in the passive circuits (not including the op-amp) are equivalent to R_f and C_f in the op-amp filter. R in this last circuit (c) is different and discussed later when appropriate.

The op-amp circuit also involves a voltage difference which powers the circuit and allows it to amplify the signal given to it.

For this experiment, we set the capacitance in all the circuits to be 100 nanoFarads. From here, we can choose different resistances to affect f_c and thereby the range of frequencies over which the phase change plays out.

The differences between low- and high-pass RC and op-amp filters are the slopes of the phase and bode plots. As will be illustrated in the subsequent section, the polarity of these slopes will vary between each circuit.

Results:

The Low-Pass RC filter

With the low-pass RC filter, we will explore two different resistances -- one at high resistance and another at a very low resistance; these being 2,200 and 50 Ω.

For each of these, we need to determine a corner frequency to know beforehand the range of frequencies we want to explore.

Starting with the higher resistance we can deduce f_c = 707 Hz. With this target frequency in mind, we can collect data to produce the plots in Figure Three.

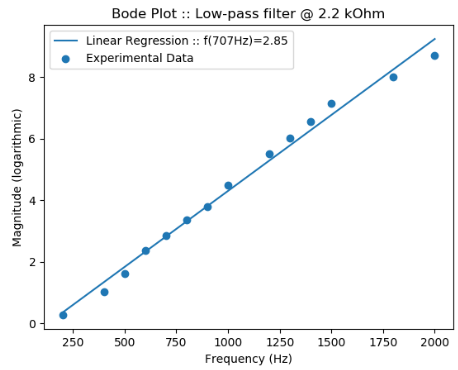
As we can see, the bode plot appears quite linear, with a consequent linear regression of r = 0.995 indicating very high correlation. The phase plot, however, takes the shape of a logarithmic curve. Regardless, each plot has a positive polarity.

The corner frequency of the 50 Ω circuit yields a much higher value f_c = 31,830 Hz.

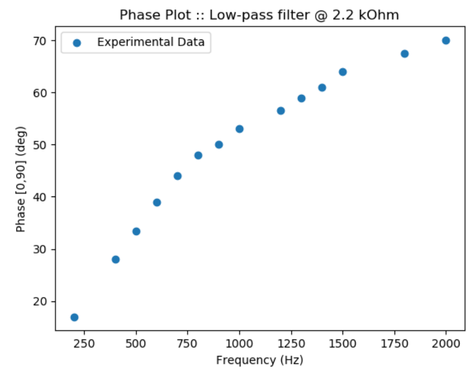
Figure Four plots this data, however it's immediately evident that the data is all over the place. This is a mistake made in the data collection stage -- a large range of frequencies are covered but fail to include high enough resolution around the corner frequency. Due to limited lab time, another set of data could not be collected.

With a rather incoherent dataset, it's impossible to draw any conclusions from these plots. Future experiments would be well-served to increase the resolution around the corner frequency -- say, in the hundreds rather than the thousands.

Figure Three: Plots for the low-pass RC filter at 2.2 kΩ

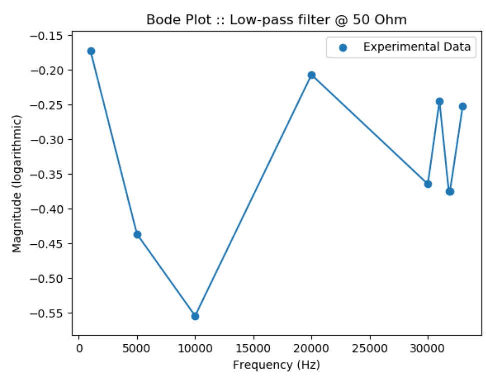


(a) Bode (magnitude) vs. frequency

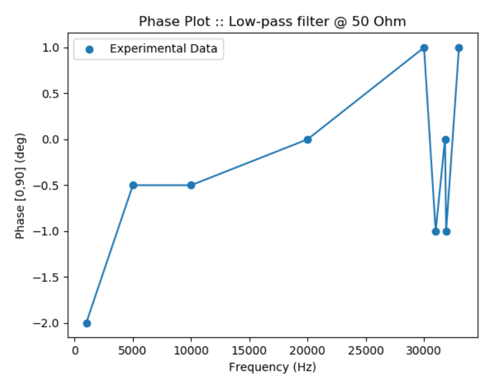


(b) Phase vs. frequency

Figure Four: Plots for the low-pass RC filter at 50 Ω



(a) Bode (magnitude) vs. frequency



(b) Phase vs. frequency

For each the bode (a) and phase (b) plots we are able to see the effect that frequency has on the relative magnitude and analog phase difference respectively. We do this for two different resistances in our low-pass RC circuit, however the lower resistance has a very high corner frequency around which the data collection was not of high enough resolution. As such, the pattern seems to break down, however we can assume that around 31,830 Hz -- within a few hundred Hz -- that the relationships would resemble Figure Three.

The High-Pass RC filter

Similar to the low-pass filter, we can choose two resistances to run through the high-pass iteration of the circuit. In our case, we choose 4,700 and 470 Ω to test this filter.

The first, 4.7 kΩ, evaluates a corner frequency f_c = 339 Hz. The data collected around this frequency gives us Figure Five.

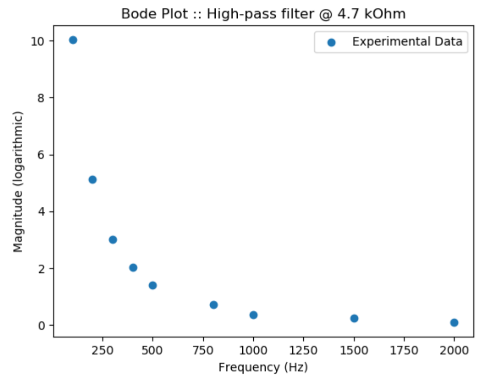
As we can see, this graph differs from Figure Three specifically in that the slopes of the high-pass filters' plots have a negative polarity. Likewise, both the bode and phase plots take on a much more exponential pattern. In this case, it appears as a decaying exponential for both of them.

The pattern continues when we plug in a 470 Ω resistor. With a corner frequency of f_c = 3.386 Hz.

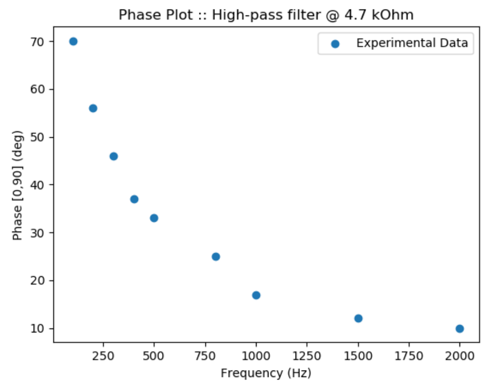
Figure Six shows the data collected for this set; almost a mirror image of the pattern shown in the higher resistance circuit but with a different range of frequencies.

Still, it's worth noting the strong similarity in the phase and bode values as the frequency increases. It suggests there is a direct proportionality; that is to say, the higher resistance 'stretches' the range over which the phase and bode plots vary.

Figure Five: Plots for the high-pass RC filter at 4.7 kΩ

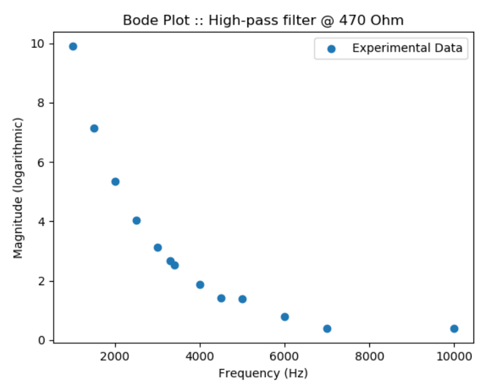


(a) Bode (magnitude) vs. frequency

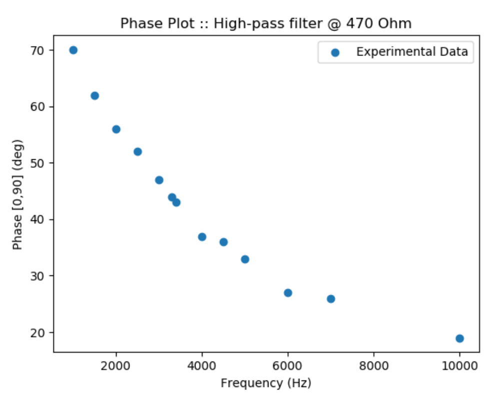


(b) Phase vs. frequency

Figure Six: Plots for the high-pass RC filter at 470 Ω



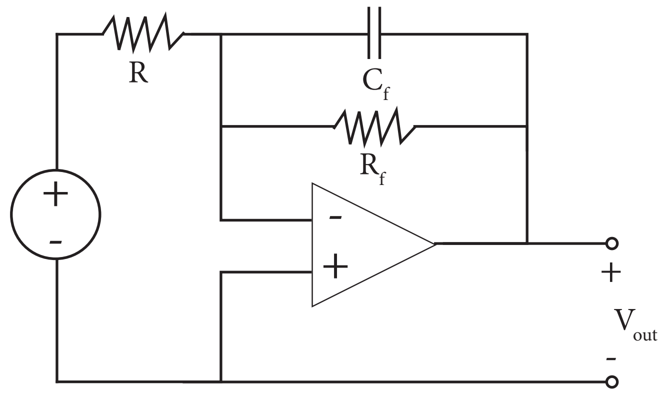
(a) Bode (magnitude) vs. frequency



(b) Phase vs. frequency

Among the bode and phase plots for this circuit, we can see a high correlation in their polarity and general steepness. It suggests that they are each factors of one another; the plot 'stretching' to fit the corner frequency depending on the resistance and capacitance.

Figure Seven: Plots for the high-pass RC filter at 4.7 kΩ



The Low-Pass Op-Amp filter

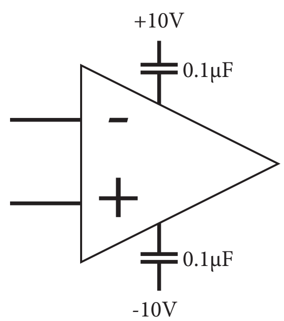
With the Op-Amp filter, we include a couple more elements in the circuit that aren't included in Figure Two (c). In order to power the op-amp element, we run a voltage across it through two capacitors, as seen in Figure Seven.

With C_f = 100 nF and our choice of a resistance R_f = 39 kΩ, we get a corner frequency f_c = 40.8 Hz. From here, we want to choose an adequate resistor for R. We want to limit the gain $G = 20 \times \log_{10}(R_f/R) < 24$ dB. With some simple algebra, we get a value R ≈ 10 kΩ.

Using this value, and choosing frequencies around f_c, we get the data which is plotted in Figure Eight.

We can easily see the specifics of how this filter differs from the others. Firstly, in contrast with the other passive circuits, the op-amp can amplify the signal, decreasing the proportionality between V_{in}/V_{out}; the bode plot starts with negative magnitude and has a positive polarity.

Likewise, the phase plot has bounds of 90 to 180 compared to the passive filters' 0 to 90 degrees. Meanwhile, in contrast to the others, the polarity of the phase plot is actually negative -- opposite to the bode plot.

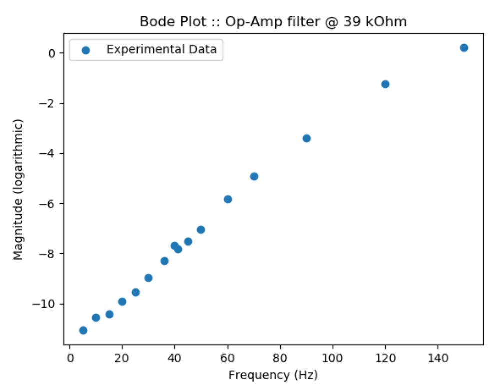


In order to power the op-amp, we need to include a potential difference bypassed by capacitors on each side.

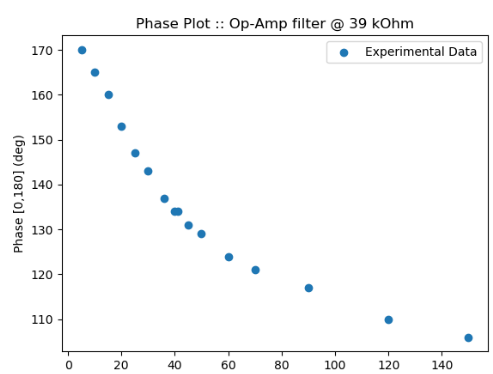
C_f remains the same as our previous filters with the addition of another resistor which moderates the gain in our system.

We choose this resistor R relative to R_f at 39 kΩ, such that R is 10 kΩ.

Figure Eight: Plots for the high-pass RC filter at 4.7 kΩ



(a) Bode (magnitude) vs. frequency



(b) Phase vs. frequency

In contrast to the other filters, the polarities of these plots are opposite one another, with the bode plot in a very linear and positive pattern and the phase plot in an expontial decay as frequency increases.

Discussion:

Having seen each of the first-order filters we explored and analyzed their bode and phase plots, we can now draw some conclusions.

The first and most important point is the breakdown of a pattern in the lower-resistance trial of the low-pass filter. Having been unable to investigate the particular corner frequency this circuit has, we cannot draw any conclusions for it. Perhaps this indicates a breakdown compared to our other filters if there is no pattern present outside the small range of values around f_c. It is difficult to say, however under more accomodating circumstances, this would have been re-investigated in the course of this report. Due to external factors, however, this was not possible. I would recommend future experiments to investigate this area more fully. For now, however, we exclude this data from our subsequent conclusions.

The second point is rather straightforward: the polarities of each of the plots. For each the low- and high-pass filters, their bode and phase plots were of the same polarity of slope. The op-amp filter, however, had slopes of opposite polarities. This is in part because of the direct function of the op-amp to amplify the input signal. As such, the bode plot starts negative and climbs to zero, as opposed to starting at zero and climbing to a maximum like the passive low-pass filter.

Lastly, the data surrounding this project lets us confirm the theory of the corner frequency around which the phase should be 45 degrees. Having seen a cohesive pattern and strong confirmation of this phenomena around each corner frequency for each plot, we can conclude that this theorem is accurate to our measurements.