Exploration of RC First-Order Time-Constant Circuits

February 18, 2020.
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Experimentation performed together with Quinn Schmidt

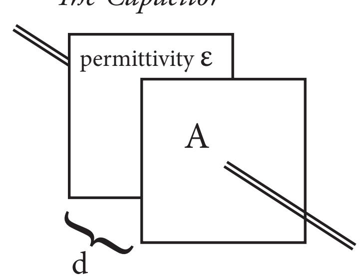
Abstract:

This report explores the behavior of RC circuits. RC circuits involve a resistor and a capacitor in series with a voltage source. The direction of the current determines it as a low- or high-pass filter with different properties for each.

These filters correspond to an exponential behavior on each rising edge of a square voltage signal, which can be paramterized and compared to experimental readings.

For the low- and high-pass filters we determined an error of roughly 7 and 40% respectively. After considering factors which could contribute to the error, we draw the conclusion that the low-pass filter agrees however the high-pass filter is inconclusive. More research is recommended to authoritatively conclude the latter.

The Capacitor



Charge q is transferred as $i = dq/dt = (\varepsilon A/d)(dv/dt)$

INRODUCTION:

Capacitors, at a fundamental level, are simply two plates of a fixed area A and separation d. Under DC circumstances, a potential difference applied across this element acts as if the capacitor is an open circuit -- a cut wire. Varying the voltage in AC conditions, however, creates a unique phenomena: the current flow becomes proportional to the derivative of the voltage with respect to time. Most importantly, a capacitor *charges* as voltage is applied across it; meaning there is a certain time before a square voltage pulse will peak.

The focus of this report is to explore the contribution of a capacitor when put into an RC circuit. This circuit includes both a resistor and a capacitor and its behavior depends on the direction of the current (see Figure One).

A current which reaches the capacitor before the resistor is called the low-pass filter. The voltage across the capacitor in each half-cycle begins at zero and logarithmically grows to peak amplitude.

Conversely, a current which reaches the resistor first defines the high-pass filter; therein, the voltage across the resistor begins at its peak and exponentially decreases to zero.

METHODS:

We are using a square-wave alternating current, which we arbitrarily set to a peak amplitude of 10V.

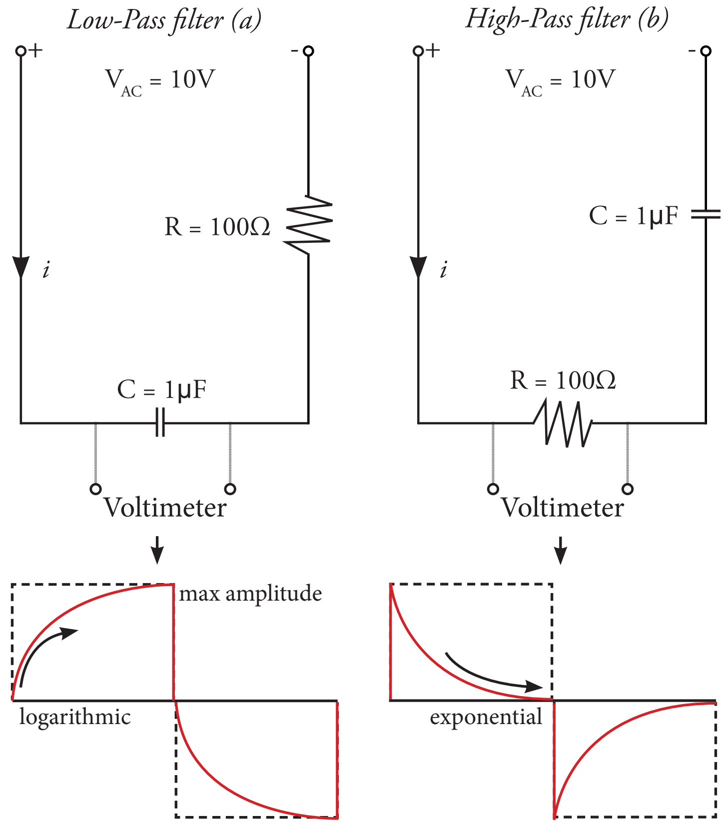
The particular qualities of a low- or high-pass filter are determined by the capacitor and resistor. The τ value parameterizes the time needed to reach exponentially proportional increments and is defined by the product of the resistance and capacitance.

Let's take the example of Figure Two for a low-pass filter. This filter will be defined by the $V_i - V_i(e^{-t/\tau})$ where V_i is the peak amplitude of the AC current. We can see that at intervals of τ, the denominators cancel and we get values 1/e, $1/e^2$ and so on to $1/e^5$. This last value reaches 99.3% of the amplitude of the function and so we can use this as a general reference point for how to determine our frequency.

In our case, both our resistor and capacitor are rated for a value $\tau = RC = (100\Omega)(1\mu F) = 1$ millisecond. A frequency of the interval needed to reach 5τ exactly would be $f_{max} = 1/5\tau = 2kHz$. We can take a frequency about an order smaller as our signal generator frequency such that $f_{SG} = 200Hz$ to ensure that we get the entire rise, peak and stabilization.

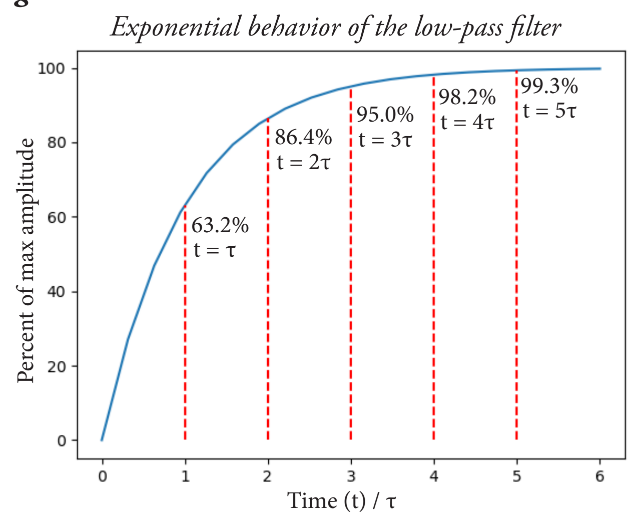
The key data to consider moving forward is our frequency $f_{SG} = 200Hz$, our square-wave AC voltage

Figure One



The schematics and correspondent waveform for the voltage read by the voltimeter are shown for the Low- and High-Pass filters.

Figure Two



The theoretical regression for a low-pass filter, whose general dependence on the τ is similar for the high-pass filter. This value corresponds to both the product of the capacitance and resistance and to the inverse exponentiation of e (e.g. $1/e^2$).

V_i = 10V and lastly the value τ = 1ms by which we will scale our time axes.

Tʜᴇ Lᴏᴡ-Pᴀss Fɪʟᴛᴇʀ:

We begin with the low-pass filter shown in Figure One (a). As we have discussed already, our function $f(t) = V_i - V_i(e^{-t/\tau})$. As such, for each integer multiple of τ, $f(n) = 10 - 10(e^{-n})$.

As with Figure Two, we can determine the theoretical voltage at each value n. We can then take our oscilloscope readings and measure the voltage across the capacitor at each increment of τ. The following table collates this data:

τ	Theoretical (V)	Experimental (V)	% Difference
1	6.32	5.43	14%
2	8.64	7.75	10%
3	9.50	8.80	7.3%
4	98.2	9.45	3.8%
5	99.3	9.85	0.81%
		average:	7.3%

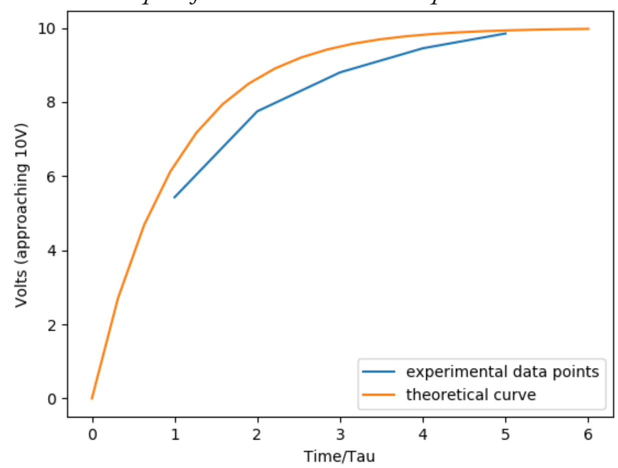
Figure Three plots this information against the curve of our theoretical function f(t).

As we would expect our error decreases as the overall voltage increases and the capacitor approaches its maximum voltage. Even so, its maximum difference reaches roughly 14% at nearly a full volt difference from theory.

Ordinarily, this would indicate a significant deviation from theory however the behavior which it very closely approaches the theoretical maximum suggests that the pattern is coherent.

Figure Three

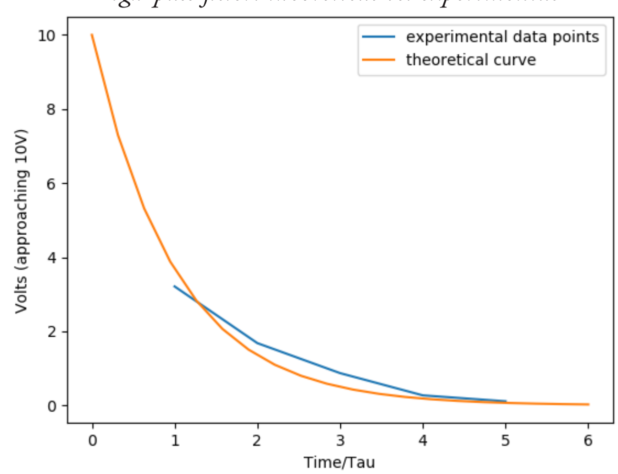
Low-pass filter: theoretical vs. experimental



This figure compares the theoretical regression of the low-pass filter to the experimental data points collected for it.

Figure Four

High-pass filter: theoretical vs. experimental



This figure compares the theoretical regression of the high-pass filter to the experimental data points collected for it.

Tʜᴇ Hɪɢʜ-Pᴀss Fɪʟᴛᴇʀ:

Our function for the high-pass filter differs in that $f(t) = V_i(e^{-t/\tau})$. Therefore each multiple of τ, $f(n) = 10(e^{-n})$.

Plugging values into our function f(n) we can find theoretical values. We can then take our oscilloscope readings and measure the voltage across the resistor at each increment of τ. The following table collates this data:

τ	Theoretical (V)	Experimental (V)	% Difference
1	3.67	3.21	12.8%
2	1.35	1.68	24.4%
3	0.50	0.87	74%
4	0.18	0.27	50%
5	0.07	0.11	64%
		average:	40%

This percentage deviation from theory appears damning, however the pattern demonstrated by Figure Four which graphs our theoretical function f(t) against our data suggests a consistent pattern approaching zero. Deviations tend to be about the theoretical predictions and those deviations are relative to the small values which we are measuring, a point that we consider more deeply in the subsequent section.

Discussion:

The possible sources of our error is the primary point to be discussed before we can consider the level of agreement our data enjoys with theory.

For each of our datasets, the error corresponds exceptionally with the actual magnitude of the data involved. The Low-Pass filter displays this exceptionally well, with smaller data points bearing greater error (e.g. 5.43V & a difference of 12.3%) whereas larger points have smaller error. Therefore part of this discrepancy can be attributed to the random error of our data collection efforts. As we collect smaller magnitude datapoints, the contribution of random error at a generally constant level of noise exacerbates the fractional difference as a proportion of a smaller value. As such, we can conclude that part of our error is the fault of random error when recording data.

While this holds true, however, random error does not assume the full fault of our error. This idea breaks down moreso around the high-pass filter dataset. Moreover we can verify its contribution by taking the raw numerical difference between our theoretical and experimental data and seeing if it is generally consistent.

For example, with the low-pass filter:

τ	Theoretical (V)	Experimental (V)	Difference
1	6.32	5.43	0.89
2	8.64	7.75	0.89
3	9.50	8.80	0.7
4	9.82	9.45	0.37
5	9.93	9.85	0.08

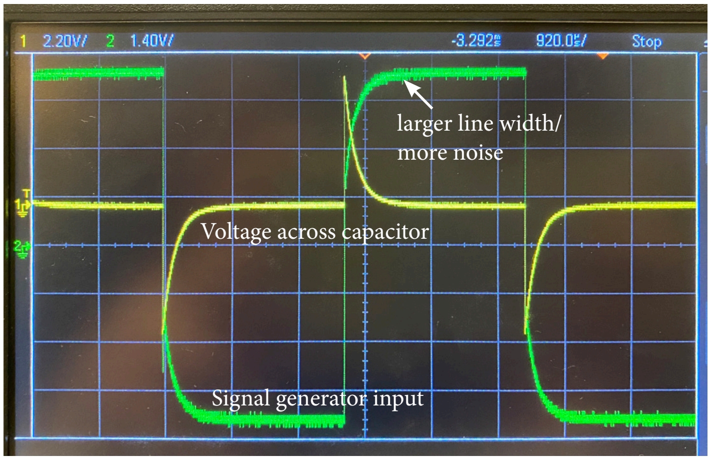
Unfortunately, it is not consistent enough. Therefore we can conclude that though it is a contributing factor, there exists another influence that works in tandem with this error.

The likeliest possibility is depicted in Figure Five, which shows the input from the signal generator (green) and the correspondent voltage change across the resistor (yellow). There is an important factor in this image: the amplitude of the noise as the wave stabilizes.

This is purely practical in that we can expect a different reading when measuring the value at the top, middle or bottom of the line. We chose to measure it from the top.

Figure Five

Oscilloscope channels across the high-pass



This image of an oscilloscope shows the waveforms for both the input signal generator and the voltage across the resistor of a high-pass filter circuit.

Lower measurements could be systematically off by, say, 0.89 volts. However, assuming the 'true' voltage lies somewhere in the middle of the line, our upper reading will continuously increase relative to the rest. As such, a phenomena where values near noisier areas of the line are shifted upwards can appear.

In sum, our error could possibly be accounted by a combination of random error and proportionalized systemic error from our oscilloscope measurements.

The question, then, of whether our measurements agree with theory is still open. From the facts, we cannot conclusively say that the data fits agreement, however relative to what we know, the error seems at least within reason.

With this motivation, this report concludes the relationship to be tenuous but reasonable for the low-pass filter and inconclusive for the high-pass filter; though I am optimistic that future experimentation will reveal a more consistent correlation for the latter.