Exploration of Fundamental Equivalent Circuits

February 4, 2020.
By Xavier Boluna

ECE 101 :: Professor Sara Abrahamsson

Experimentation performed together with Quinn Schmidt

Abstract:

This report explores equivalent circuits and their properties. The Thevenin and Norton theories for equivalent circuits are applied to a T-Network. Experimental data agrees with theory significantly to a less-than 3% fractional difference.

Turning towards ideal power source, this report investigates a 10V DC source and determines its equivalent Thevenin resistance and the error it introduces when connected to a load resistor in parallel. Data shows a consistent 1.2% error across the voltage and current, which can be considered a negligible but propapagable known error when using the source.

Lastly, we tested the maximum power transfer theory using our T-Network across a load resistor and calculated an average deviation from theory of 2.5%.

Inroduction:

An electric outlet is rated at a certain current, wattage and voltage. The actual path which the electricity takes is long, complicated and influenced by a variety of factors, elements and devices; yet, when you plug in your lamp, the output attributes are easily known. Gigawatts of power at the plant, miles of wiring and the contents of its circuit can be boiled down to an equivalent circuit.

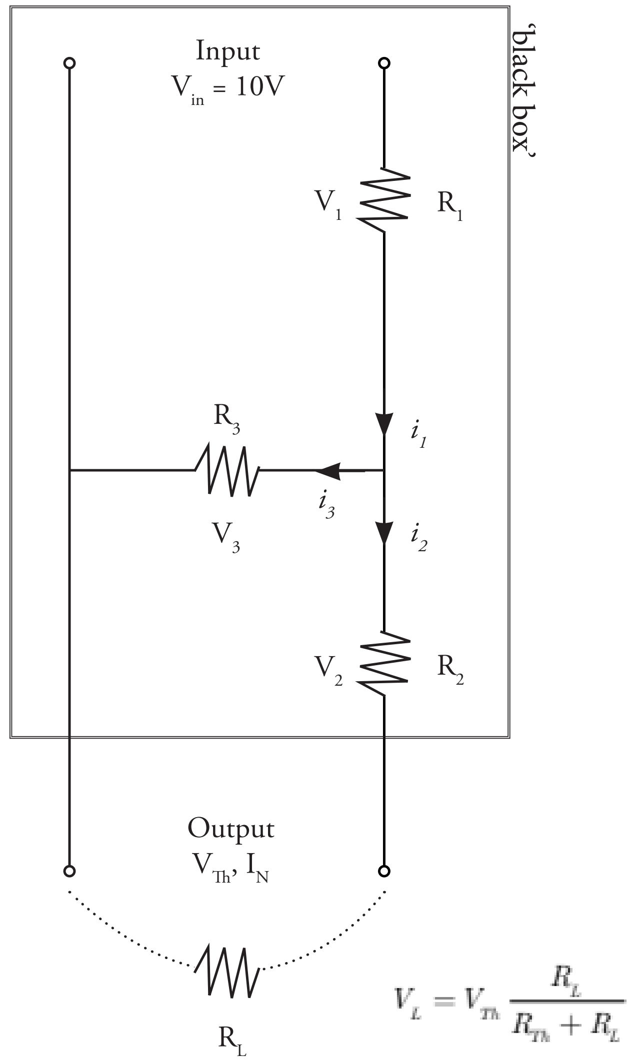
Such is the purpose of Thevenin and Norton's theories for equivalent circuits. They allow one to treat a circuit whose outputs can be described as the equivalent of a resistor and voltage source in series, or a resistor and current source in parallel.

Treating an equivalent circuit as a source invites analysis of how ideally it behaves, and introduces different behaviors for power transfer.

These ideas are explored in this report, starting first with equivalent circuits' properties and their relationship with the load resistor, then with non-ideal power sources and, lastly, with maximum power transfer.

Figure One

The T-Network



Thevenin and Norton Equivalent Circuits:

The circuit for which we will find an equivalent circuit is the T-Network, shown in Figure One, complete with three 100Ω resistors and driven by a DC potential difference of 10V with a current of 66mA. To find the equivalent circuit, we pretend the power source and circuit are contained in a 'black box,' which represents the properties contained within the circuit.

The first and potentially most useful metric we can measure from this mystery box is its equivalent resistance, or the Thevenin Resistance R_{Th}. We obtain this by switching off the power supply and calculating the equivalent resistance of the circuit with no current through it. Operating on Figure One to find the equivalent resistance, we combine R_1 and R_3 in parallel with R_2 in series and obtain $R_{Th} = (R_1 \| R_3) + R_2 = 150\Omega$. We can then measure the resistance across our output terminals with our power source switched off using an ohmmeter and obtain an experimental value of 148.2Ω -- a fractional difference of just 1.2%.

Knowing both our Thevenin resistance and input voltage, we can calculate using the voltage divider theorem across an arbitrary load resistor. For simplicity's sake, we choose a 100Ω resistor, making our equation for the open circuit voltage $V_{OC} = R_L / (R_{Th} + R_L) \times V_{in} = 100 / 200 \times 10 = 5V$. We measure find this potential difference to be 4.98V; off by less than a percent.

Likewise it becomes easy to calculate our equivalent current. Knowing our V_{Th} and R_{Th}, $I_N = V_{Th}/R_{Th} = 33.3$ mA. We find our experimental value to be 32.5mA, with a difference of 2.4%.

Perhaps most important is ensuring that, for edge cases, the theory holds true. For this reason, we implement a potentiometer in the place of our fixed-resistance load resistor. We can then vary this potentiometer and evaluate the correspondent theoretical and experimental values for voltage and current.

One important methodological note is that reading the resistance of a potentiometer when it is placed in a circuit will noy read correct values. The voltage drop measured across the resistor is measured also across the 'parallel' circuit around the resistor. For that reason, whenever we change its rating, we measure it outside of the circuit before placing it into the circuit.

We chose ratings of 10 kΩ and 600Ω. Using the same equations as before:

10 kΩs			
Value	Theoretical	Experimental	Differ-ence
V_L	$V_{Th} \times R_L/(R_L + R_{Th})$ = 4.93V	4.97V	0.8%
I_L	$V_{Th} / (R_{Th} + R_L)$ = 0.492mA	0.488mA	0.8%

600 Ωs			
Value	Theoretical	Experimental	Differ-ence
V_L	$V_{Th} \times R_L/(R_L + R_{Th})$ = 4.00V	3.99V	0.3%
I_L	$V_{Th} / (R_{Th} + R_L)$ = 6.66mA	6.56mA	1.5%

As we can see, every value we test agrees very well.

Non-ideal Power Sources:

One element we have used thus far is, itself, an equivalent circuit which we treat ideally. I speak, of course, of the power source for the circuit V_{in}. As we now know, it is a flawed premise to assume it to be ideal; we must evaluate how ideal this source is in order to go forward comfortably with our assumptions.

Using an additional 100Ω resistor in parallel with the power source, we can measure using our multimeter the Thevenin resistance R_{Th} = 1.45Ω and set voltage V_{Th} = 10V. With our known equations, we can compare theory with practice:

Non-ideal power source (R_L = 100Ω)			
Value	Theoretical	Experimental	Differ-ence
V_L	$V_{Th} \times R_L/(R_L + R_{Th})$ 9.86V	9.98V	1.2%
I_L	$V_{Th} / (R_{Th} + R_L)$ = 98.6mA	97.4mA	1.2%

Apparently, there is a consistent error within the power source of roughly 1.2%. We know this is consistent as the proportional relationship between voltage and current implies that a 1.2% increase in V_L must correspond to a 1.2% decrease in I_L if the resistances remain the same.

Regardless, this small of an error has a minimal contribution to most calculations and so, with this element of systemic error in mind should be encounter more significant error, we can consider this contribution to be negligible. The power source is relatively ideal.

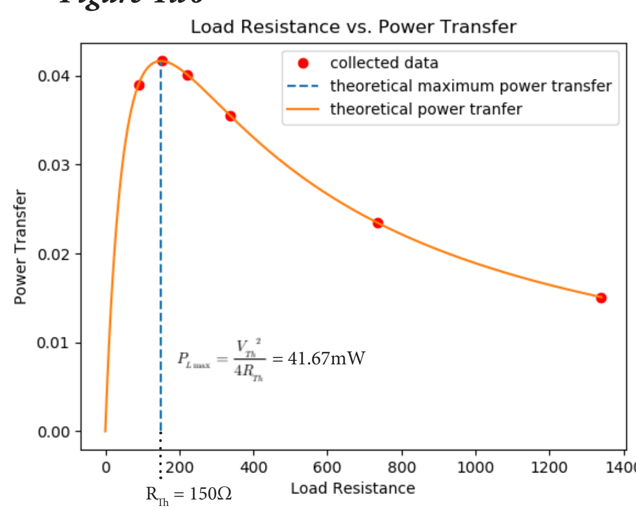
Maximum Power Transfer:

For a Thevenin or Norton equivalent circuit, there exists a theoretical maximum power transfer possible across the load resistor. This is maximum is reached when the load resistor's value is equal to R_{Th}. The theoretical maximum power transfer is thusly defined by the equation shown in Figure Two, and comes out to 41.67mW.

Figure Two collates data from Table One centered around our known Thevenin resistance R_{Th} = R_L = 150Ω. As we can see, a point very near that value forms a distinguishable peak. The function apparently increases rather quickly and diminishes slowly as R_L grows large.

We can determine fractional error by calculating the theoretical power transfer for every R_L which we recorded. The average of these is 2.5%, suggesting a strong agreement between the theoretical power transfer (orange in Figure Two) and our collected data.

Figure Two



This graph compares the collected data computed for power transfer(from R_L and V_L) against the theoretical expected values. The data differs on average 2.5% from the theoretical power transfer.

Theory mandates that the peak be at $R_L = R_{Th}$ which, in our case, rests at 150Ω. We observe this to be true.

Discussion:

This report requires little more discussion; a testament to the strength of the theoretical claims which we tested.

Our equivalent circuits in the first section functioned as intended, and we were able to create an equivalent Thevenin and Norton circuit for our T-Network. Our calculated values for each of the elements was in good agreement.

Testing our power source yielded a small and consistent fractional error for our power source, allowing us consider its effect if needed to in the future. That being said, its effect appears largely negigible.

Lastly, we tested the maximum power transfer theory on our T-Network and compared the voltage across different load resistors.

Altogether, all our tests are in extremely good agreement, with no greater than a 3% fractional difference from theory across all experimental data. This difference can be attributed partially to the intrinsic resistivity of copper wires, and to random error. Additionally, we can now consider part of this error to be the fault of the power source, whose output is non-ideal and therefore in some way culpable for error.

Table One

Trial	Load Resistance (Ω)	Voltage (V)	Power Transfer (mW) $P_L = V_{Th}^2 R_L / (R_L + R_{Th})^2$
1	152.3	2.42	41.66
2	1,340	4.50	15.09
3	89.79	1.81	39.04
4	338.1	3.49	35.48
5	220.2	2.94	40.17
6	736.0	4.15	23.44