Verification of the Basic Laws of Electronic Circuits

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ECE 101 :: Professor Sara Abrahamsson

Experimentation performed together with Quinn Schmidt

Fundamentally, the study of electronic circuits rests on several key mathematical relationships which describe the interactions between current, voltage and resistance. This report tests these relationships and their corollaries across several circuits designs, including a T-network and a Wheatstone Bridge.

In the first section, we will verify Kirchoff's Voltage & Current Laws, Ohm's Law and the Current & Voltage Divider Theorems.

In the second, we treat the differences between single-ended, differential and common-mode voltages.

Lastly, we describe the property of superposition and their relationship with linearity.

Verification of basic laws & derived quantities.:

Let's describe our first basic circuit: the T-Network. Shown in Figure One, the T-Network is composed of a single resistor in series with two additional resistors in parallel. An input voltage is drawn across the first two open terminals.

For the purposes of our experiment, we elected to use resistors of equal resistance: $R_1 = R_2 = R_3 = 100\Omega$.

Our first goal is to consider Kirchoff's Current and Voltage Laws. The former is defined at a node and expects for all the currents entering it to equal to those exiting it. This is to say, in Figure One, if i_1 enters n_1 and i_2 and i_3 exit, we can expect $i_1 - i_2 - i_3 = 0$.

In order to measure the respective currents across the node, we use an ammeter (as shown in Figure Two).

Following this guideline, we run a voltage of 10V across the system and determine:

i_1	64.9 mA
i_2	32.7 mA
i_3	32.8 mA
$\Sigma i = i_1 - i_2 - i_3$	-0.6 mA ≈ 0

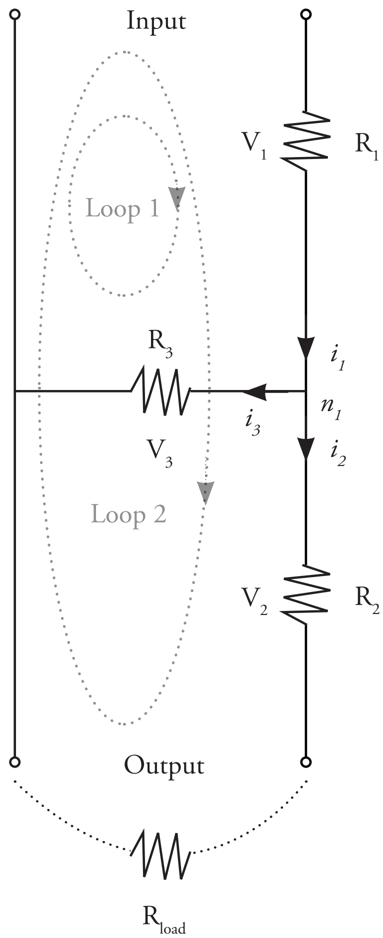
notably, our currents sum to just under a milliAmp, amounting to a roughly 1% error from our input current i_1. For this reason, we can accept this margin as the result of random error.

Next, we can test Kirchoff's Voltage Law. This law operates across 'loops' with reference to the input voltage. Across Loop 1 (in Figure One), for example, the law functions such that $V_{Input} = V_1 + V_3$; ensuring essentially that voltage is conserved. Similarly across Loop 2, $V_{Input} = V_1 + V_2$.

Each element -- or in our case, resistor -- produces a voltage drop across its terminals which can be measured with a voltimeter connected in parallel (see Figure Three).

The T-Network

Figure One



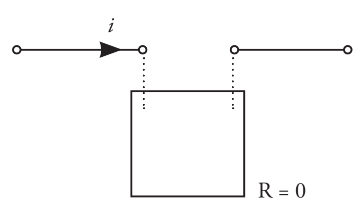
Ohm's Law:
$V = iR$
e.g. $V_1 = i_1 R_1$

Kirchoff's Current Law:
$\Sigma i = 0$
(all currents incident and emitted on a node sum to zero.)

Kirchoff's Voltage Law:
$\Sigma V = 0$
(all currents within a loop sum to zero.)

Figure Two

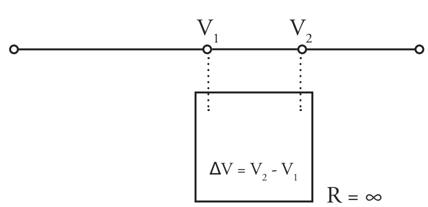
The Ammeter



In order to measure current, the ammeter must be connected in series to the section from which the reading is desired.
An ideal ammeter has zero resistance.

The Voltimeter

Figure Three



In order to measure voltage, the voltimeter must be connected in parallel. In this way, we can measure the difference in voltage across each terminal.

For this reason, an ideal voltimeter has infinite resistance.

Running again, a voltage of 10V across the input, we determine:

V_1	6.65V
V_2	3.34V
V_3	3.34V
loop 1: $V_{input} - V_1 - V_3$	0.04V ≈ 0
loop 2: $V_{input} - V_1 - V_2$	0.04V ≈ 0

Again, as our error amounts to just 0.4% of our input voltage, we can consider this random error.

Another fundamental law of circuitry is the conservation of Power, or the Work per unit time. Power is defined as the product of Voltage and Current ($P = iV$), which means we can determine this value using the data we have already collected. Power (on a non-negligible scale) is released or emitted by the elements across a circuit -- in our case, the resistors and signal generator.

$P_{input} = i(V_{input})$	(9.99 V)(64.9 mA) = 648 mW
P_1	434 mW
P_2	109 mW
P_3	108 mW
$\Sigma P = P_{input} - P_1 - P_2 - P_3$	3 mW ≈ 0

Our error here is actually 0.4% from the input power, again falling within the bounds of random error.

As such, we are able to confidently verify Kirchoff's Voltage & Current Laws in addition to Conservation of Power in our practical tests to a certainty of up to 1%.

Another worthwhile venture is to measure the experimental resistance of the resistors we have been using. This is easily possible using Ohm's Law. Again, we have already measured current and voltage for each of the elements and so can easily solve for $R = V/i$. Note that the listed resistance for each of these is 100 Ω.

R_2	102 Ω	error = 2%
R_3	103 Ω	3%
R_1	102 Ω	2%

Here, we see our error finally exceed 1%. It is, however, normal to expect some variation. These imperfections are unlikely to have impacted our calculations for Kirchoff's Laws or that of Conservation of Power.

Next, using these values, we can compute the equivalent resistance of the circuit to the experimental reading we obtain.

Our second and third resistors are in parallel so their equivalent resistance is represented by $(R_2 \times R_3)/(R_2 + R_3) = 51\ \Omega$. This equivalent resistance is in series with our first resistor, so the total resistance of the circuit is 102 Ω + 51 Ω = 153 Ω.

For our experimental reading, $R_{exp} = (9.99\ V)/(64.9mA) = 154\ \Omega$. We can thusly conclude that our 0.7% difference is the result of random error.

Lastly, for this circuit, we explore the Current & Voltage Divider Theorems.

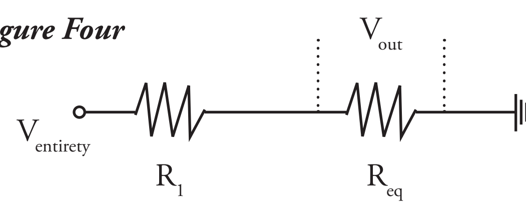
The Current Divider Theorem argues that, following Figure One: $i_2 = (i_1 R_3) / (R_2 + R_3)$ and $i_3 = (i_1 R_2) / (R_2 + R_3)$. This is easy to check with our recorded values. As we know $R_2 \approx R_3$, we simplify $i_2 = i_3 = i_1\ (100/200) = 32.5$ mA.

As we have already recorded measurements for i_2 and i_3, we can compare them to see an error of up to 1%.

The Voltage Divider Theorem requires that we simplify our diagram to two resistor elements in series. Similarly to how we did before for R_2 and R_3, $R_{eq} = 50\ \Omega$.

The voltage differential across the second resistor $V_{out} = (V_{entirety}\ R_{eq}) / (R_1 + R_{eq})$.

Figure Four



The Voltage Divider Theorem encapsulates the voltage across an element within series circuit.

Using data from the previous experiments, we can determine $V_{out} = (9.99V)(50/150) = 3.33$ V. This value conforms to our readings of $V_2 = V_3 = 3.34$ V wherein voltage does not divide between parallel branches. The error here is just 0.3%.

Single-ended & Differential Circuits:

This experiment's basis is the Wheatstone Bridge, pictured in Figure Five (a). Note that R_4 is a variable resistor -- or, in our case, a potentiometer. The actual creation of this schematic can vary largely and (b) is a labeled picture of our own version of this circuit.

Consider $V_d = V_{23}$ to be where the voltimeter is linked up, measuring the voltage differential between each arm. The differential voltage V_d is defined by the relationship $V_2 - V_3$. The common-mode voltage follows $V_{cm} = (V_2 + V_3)/2$.

In the first trial, we dialed the potentiometer (R_4) to its highest resistance and ran 10 V through the circuit:

V_2	5.01 V
V_3	9.86 V
experimental V_d	-4.84 V
$V_d = V_2 - V_3$	-4.85 V
$V_{cm} = (V_2 + V_3)/2$	7.44 V

wherein it is immediately apparent that the relationship described for V_d is accurate to the theoretical measurement to a 0.2% difference.

Following in the second trial with the lowest resistance possible:

V_2	5.00 V
V_3	35 mV
experimental V_d	4.96 V
$V_d = V_2 - V_3$	4.97 V
$V_{cm} = (V_2 + V_3)/2$	2.52 V

Again, we see that the differential voltage differs by just 0.2%.

This matches our understanding of single-ended voltages and differential voltages. A single-ended voltage difference is like that measured at nodes 2 and 3 -- their respective voltage drops are independent of the other.

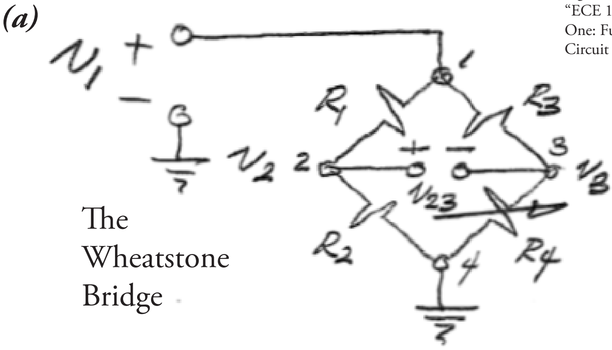
The differential voltage, however, is intrinsically dependent by the other. It is the difference and, as in the case of the first trial, a high resistance in the 'right' branch forces the majority of the voltage to the 'left branch.' This is why V_d becomes negative, as $V_3 > V_2$. The opposite is true for the second trial.

The common-mode voltage describes the average of the two single-ended voltages. The variation therein is relevant to their relationship with R_4. For example, V_2 is connected in series and so bears no change from one re-

Figure Five

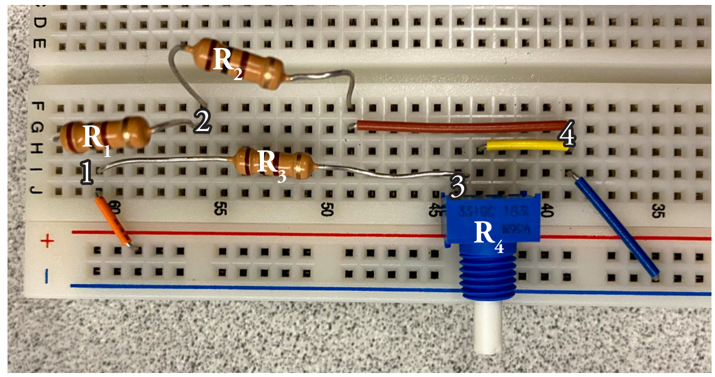
(a)

Figure source: "ECE 101 Laboratory One: Fundamental Circuit Theory Laws"



The Wheatstone Bridge

(b)



The voltimeter is attached to points 2 and 3 to measure the voltage drop V_{23} between each arm of the bridge. Each of the resistors is rated at 100 Ω.

sistance to another. V_3, however, is connected and thusly experiences a drastic change -- experiencing the largest voltage drop in the branch when R_4's resistance is large and conversely the smallest drop when its resistance is small. The common-mode voltage thusly measures about half the total input voltage (10 V / 2 = 5 V) for the former and about half of the voltage drop across V_2 (5 V / 2 = 2.5 V) when V_3's drop is negligible for the latter.

We can sum this up by describing the differential voltage as dependent on the ratio of the bridge resistors V_2:V_3. As V_d becomes negative when $V_3 > V_2$ and positive when $V_3 < V_2$.

Meanwhile, the common-mode voltage depends on each independent voltage reading across a branch.

It's now also worth verifying the experimental resistances for each of the consitituent resistors R_1, R_2 and R_3. We can do this by measuring their respective currents and voltage drops. Running 10 V through the circuit and leaving the potentiometer on its highest resistance:

$V_1 = 4.90$ V	$I_1 = 48.9$ mA	$R_1 = 100$ Ω
$V_2 = 5.00$ V	$I_2 = 48.9$ mA	$R_2 = 102$ Ω
$V_3 = 9.93$ V	$I_3 = 96$ mA	$R_3 = 103$ Ω

giving us a maximum difference of 3% for R_3.

NON-LINEAR RESISTANCES:

In the course of exploring resistors, it becomes necessary to address superposition. In the simplest terms, superposition is the mathematical law that every change in voltage expects a correspondent change across the elements of a circuit For example, doubling $2V_A = 2(V_B + V_C) = 2\ V_B + 2\ V_C$.

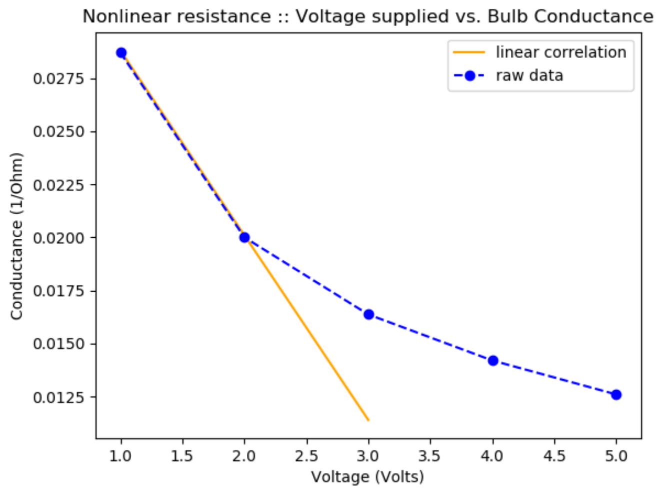
In order to explore this property, we ran differing voltages across each a resistor and bulb to measure their correspondent change in current and, thereby, conductance.

Figure Six depicts the data collected in Table One and clearly shows a linear relationship between the voltage and conductance of the resistor (red). A linear regression is fitted onto the data (green) which shows the pattern generated by the data and allows us to determine a goodness-of-fit coefficient, R^2 = (0.997)^2 ≈ 1% disagreement. This suggests the data is extremely comformative to its linear pattern.

Figure Seven depicts the data collected in Table Two and shows the relationship the conductance of the bulb shares with different voltages. As is immediately evident, the conductance varies much more widely with each change in voltage. Additionally, the relationship is negative -- meaning that each additional increment in voltage is met with a lower conductance.

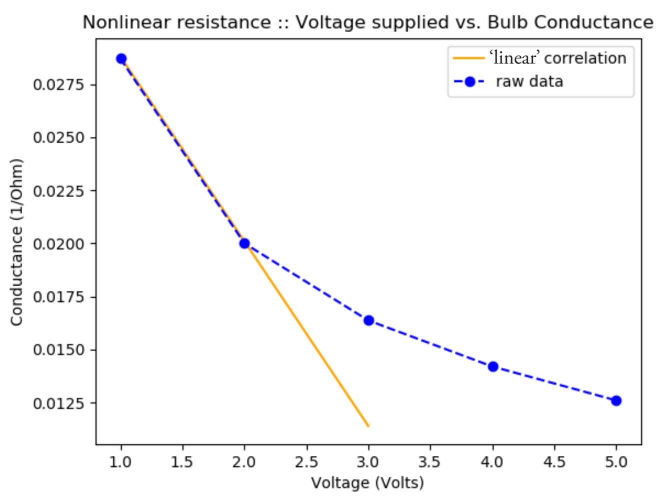
As such, we can conclude that the resistor obeys superposition due to its linearity in Figure Six whereas the bulb does not.

Figure Six



The conductance of the resistor increases steadily with each increment in voltage.

Figure Seven



The conductance of the bulb increases in smaller amounts for each additional increment in voltage.

Table One

Voltage (V)	Current (mA)	Resistance (Ω)	Conductance m(1/ Ω)
1	28.7	34.8	28.7
2	40.0	50.0	20.0
3	49.1	61.1	16.4
4	56.8	70.4	14.2
5	63.0	79.4	12.6

Table Two

Voltage (V)	Current (mA)	Resistance (Ω)	Conductance m(1/ Ω)
1	9.47	106	9.47
2	18.9	106	9.48
3	28.5	105	9.50
4	38.0	105	3.80
5	47.6	105	9,52

Conlusion:

This report's exploration of the different fundamental qualities and properties of circuits verified all its experiments, to an error within 5%.

The first section dealt with Kirchoff's Laws to determine voltages around a circuit's loops and currents at each node. It verified the principle of conservation of power and used Ohm's Law to determine the resistances of different resistors. Finally, it verified the Current & Voltage Divider theorems.

The second section used a Wheatstone bridge to compare single-ended, differential and common-mode voltages. It found that the differential voltage is determined by the ratio between the single-ended voltages, whereas the common-mode voltage depends moreso on each individual branch's case.

Lastly, we used a bulb and a resistor to explore the property of superposition in a circuit and verified the linearity of a resistor and the nonlinearity of a bulb. Linearity ultimately determines whether the element obeys superposition.